2. (3,0) Seja $f(x,y) = \begin{cases} \dfrac{\cos(x^2+y^2)-1}{x^2+y^2} & \text{se } (x,y) \neq (0,0) \\ 0 & \text{se } (x,y) = (0,0) \end{cases}$

(a) Calcule $\dfrac{\partial f}{\partial x}(x,y)$, para todo $(x,y) \in \mathbb{R}^2$.

(b) É $\dfrac{\partial f}{\partial x}$ contínua em $(0,0)$?

(c) Mostre que f é diferenciável em $(0,0)$.

(a) se $(x,y) \neq (0,0)$,

$$\frac{\partial f}{\partial x}(x,y) = \frac{-2x\,(x^2+y^2)\,\text{sen}(x^2+y^2) - 2x.\left[\cos(x^2+y^2)-1\right]}{(x^2+y^2)^2}$$

$$\frac{\partial f}{\partial x}(0,0) = \lim_{h\to 0} \frac{f(0+h,0)-f(0,0)}{h} = \lim_{h\to 0} \frac{\cos h^2 - 1}{h^3} =$$

$$\underset{\text{L'Hopital}}{=} -\lim_{h\to 0} \frac{2h.\,\text{sen}\, h^2}{3h^2} = -\frac{2}{3}\underbrace{\left(\lim_{h\to 0} h\right)}_{=0}.\underbrace{\left(\lim_{h\to 0}\frac{\text{sen}\, h^2}{h^2}\right)}_{=1} = 0$$

(b) $$\frac{\partial f}{\partial x}(x,y) = -2x\,\underbrace{\frac{\text{sen}(x^2+y^2)}{x^2+y^2}}_{\xrightarrow{(x,y)\to(0,0)}\ 1} - 2x\,\underbrace{\frac{\cos(x^2+y^2)-1}{(x^2+y^2)^2}}_{\to\ ?}$$

$$\lim_{(x,y)\to(0,0)} \frac{\cos(x^2+y^2)-1}{(x^2+y^2)^2} = \lim_{u\to 0}\frac{\cos u - 1}{u^2} \underset{\text{(L'Hopital}}{=} -\lim_{u\to 0}\frac{\text{sen}\, u}{2u} =$$

Logo, $\displaystyle\lim_{(x,y)\to(0,0)} \frac{\partial f}{\partial x}(x,y) = -2.0.1 - 2.0.\left(-\frac{1}{2}\right) = 0 = \frac{\partial f}{\partial x}(0,0)$

Logo, $\dfrac{\partial f}{\partial x}$ é contínua em $(0,0)$.