

$$\lim_{(x,y) \rightarrow (0,0)} \frac{xy}{x^2+y^2} \cdot \sin\left(\frac{xy^2}{x^2+y^2}\right) = 0.$$

↑ limitada
↓ 0

(b) Seja  $f(x,y) = \frac{xy^2}{x-y}$

$$f(x,y) = 1 \iff xy^2 = x - y \iff x = \frac{y}{1-y^2}$$

Seja  $\gamma(t) = \left(\frac{t}{1-t^2}, t\right)$ .  $\gamma(0) = (0,0)$  e

$$f(\gamma(t)) = 1 \quad \therefore \quad \lim_{t \rightarrow 0} f(\gamma(t)) = 1$$

Por outro lado, seja  $\mu(t) = (t, 0)$ .  $\mu(0) = (0,0)$  e

$$f(\mu(t)) = \frac{0}{t} = 0 \quad \cdot \quad \text{Logo}$$

$$\lim_{t \rightarrow 0} f(\mu(t)) = 0 \neq 1 = \lim_{t \rightarrow 0} f(\gamma(t))$$

Logo, o limite pedido não existe