

4. (a) (1,0) Seja $f(x) = \arcsen(\sqrt{1-x^4})$. Calcule $f'(0)$.

(b) (1,0) Seja $g(x) = (2 + \cos x)^{1/x^2}$. Calcule $g'(x)$.

$$a) \lim_{x \rightarrow 0} f(x) = \lim_{x \rightarrow 0} \arcsen(\sqrt{1-x^4}) = \arcsen(1) = f(0)$$

$$\therefore \lim_{x \rightarrow 0} [f(x) - f(0)] = 0$$

$$f'(0) = \lim_{x \rightarrow 0} \frac{f(x) - f(0)}{x - 0} \stackrel{L'H}{=} \lim_{x \rightarrow 0} \frac{f'(x)}{1} =$$

$$= \lim_{x \rightarrow 0} \frac{1}{\sqrt{1 - (\sqrt{1-x^4})^2}} \cdot \frac{-4x^3}{2\sqrt{1-x^4}} =$$

$$= \lim_{x \rightarrow 0} \frac{1}{\sqrt{x^4}} \cdot \frac{-2x^3}{\sqrt{1-x^4}} = \lim_{x \rightarrow 0} \frac{-2x}{\sqrt{1-x^4}} = 0$$

$$b) \quad g(x) = (2 + \cos x)^{1/x^2} = e^{\frac{\ln(2 + \cos x)}{x^2}}$$

$$g'(x) = e^{\frac{\ln(2 + \cos x)}{x^2}} \cdot \frac{1}{x^4} \left[\frac{-\sin x}{2 + \cos x} \cdot x^2 - 2x \ln(2 + \cos x) \right]$$