3. Calcule

(a) (1,5) $\int x\cos^3 x\,\mathrm{sen}x\,\mathrm{d}x$

(b) (1,5) $\int_2^3 \frac{x+1}{\sqrt{x^2-4x+5}}\mathrm{d}x$

a) $\int \underbrace{x}_{g(x)} \underbrace{\cos^3 x \,\mathrm{sen}\, x}_{f'(x)}\, dx = x\left(\frac{-\cos^4 x}{4}\right) - \int 1\left(\frac{-\cos^4 x}{4}\right) dx$

$\cos^4 x = (\cos^2 x)^2 = \left(\frac{1+\cos(2x)}{2}\right)^2 = \frac{1}{4}\left(1 + 2\cos(2x) + \cos^2(2x)\right) =$

$= \frac{1}{4}\left(1 + 2\cos(2x) + \frac{1+\cos(4x)}{2}\right) =$

$= \frac{1}{4}\left(\frac{3}{2} + 2\cos(2x) + \frac{\cos(4x)}{2}\right)$

$\therefore$

$\int x\cos^3 x\,\mathrm{sen}\,x\,dx = \frac{-x\cos^4 x}{4} + \int \frac{1}{16}\left(\frac{3}{2} + 2\cos(2x) + \frac{\cos(4x)}{2}\right) dx$

$= \frac{-x\cos^4 x}{4} + \frac{3}{32}x + \frac{1}{16}\mathrm{sen}(2x) + \frac{1}{128}\mathrm{sen}(4x) + K$

b) $$\int_2^3 \frac{x+1}{\sqrt{x^2-4x+5}}\,dx = \int_2^3 \frac{x+1}{\sqrt{(x-2)^2+1}}\,dx$$

$$x = 2+\operatorname{tg} t \qquad t \in \left]-\frac{\pi}{2}, \frac{\pi}{2}\right[$$

$$dx = \sec^2 t\,dt$$

$$t=0 \Rightarrow x=2$$

$$t=\frac{\pi}{4} \Rightarrow x=3$$

$$\int_2^3 \frac{x+1}{\sqrt{x^2-4x+5}}\,dx = \int_0^{\pi/4} \frac{3+\operatorname{tg} t}{\sec t}\sec^2 t\,dt =$$

$$= \int_0^{\pi/4} 3\sec t\,dt + \int_0^{\pi/4} \operatorname{tg} t \sec t\,dt =$$

$$= 3\ln(|\sec t + \operatorname{tg} t|)\Big|_0^{\pi/4} + \sec t\Big|_0^{\pi/4} =$$

$$= 3\ln(\sqrt{2}+1) - 3\ln(1+0) + \sqrt{2} - 1 =$$

$$= 3\ln(\sqrt{2}+1) + \sqrt{2} - 1$$