3. Calcule

(a) (1,5) $\int x \operatorname{sen}^3 x \cos x \, dx$

(b) (1,5) $\int_1^2 \frac{x+1}{\sqrt{x^2-2x+2}} dx$

a) $\int \underbrace{x}_{g(x)} \underbrace{\operatorname{sen}^3 x \cos x}_{f'(x)} dx = x \frac{\operatorname{sen}^4 x}{4} - \int 1 \cdot \frac{\operatorname{sen}^4 x}{4} dx$

$\operatorname{sen}^4 x = (\operatorname{sen}^2 x)^2 = \left(\frac{1-\cos(2x)}{2}\right)^2 = \frac{1}{4}\left(1 - 2\cos(2x) + \cos^2(2x)\right) =$

$= \frac{1}{4}\left(1 - 2\cos(2x) + \frac{1+\cos(4x)}{2}\right) =$

$= \frac{1}{4}\left(\frac{3}{2} - 2\cos(2x) + \frac{\cos(4x)}{2}\right)$

$\therefore$

$\int x \operatorname{sen}^3 x \cos x \, dx = \frac{x \operatorname{sen}^4 x}{4} - \int \frac{1}{16}\left(\frac{3}{2} - 2\cos(2x) + \frac{\cos(4x)}{2}\right) dx$

$= \frac{x \operatorname{sen}^4 x}{4} - \frac{3}{32} x + \frac{\operatorname{sen}(2x)}{16} - \frac{\operatorname{sen}(4x)}{128} + K$

3 A

b) $$\int_1^2 \frac{x+1}{\sqrt{x^2-2x+2}}\,dx = \int_1^2 \frac{x+1}{\sqrt{(x-1)^2+1}}\,dx$$

$x = 1 + tgt \quad t \in \left]-\frac{\pi}{2}, \frac{\pi}{2}\right[$

$dx = \sec^2 t \, dt$

$t = 0 \Rightarrow x = 1$

$t = \frac{\pi}{4} \Rightarrow x = 2$

$$\int_1^2 \frac{x+1}{\sqrt{x^2-2x+2}}\,dx = \int_0^{\pi/4} \frac{2+tgt}{\sec t}\cdot \sec^2 t\,dt =$$

$$= \int_0^{\pi/4} 2\sec t\,dt + \int_0^{\pi/4} tgt\sec t\,dt =$$

$$= 2\ln(|\sec t + tgt|)\Big|_0^{\pi/4} + \sec t\Big|_0^{\pi/4} =$$

$$= 2\ln(\sqrt{2}+1) - 2\ln(1+0) + \sqrt{2} - 1 =$$

$$= 2\ln(\sqrt{2}+1) + \sqrt{2} - 1$$