

Questão 1 (Turma B)

Item a) Aplicamos duas vezes a integração por partes:

$$\begin{aligned}\int x^3(\ln(x))^2 dx &= \frac{x^4}{4}(\ln(x))^2 - \frac{1}{2} \int x^3 \ln(x) dx \\ &= \frac{x^4}{4}(\ln(x))^2 - \frac{1}{2} \left(\frac{x^4}{4} \ln(x) - \frac{1}{4} \int x^3 dx \right) \\ &= \frac{x^4}{4}(\ln(x))^2 - \frac{1}{8} x^4 \ln(x) + \frac{1}{32} x^4 + c\end{aligned}$$

Item b) Usamos a substituição $u = x^4$ e, em seguida, $v = \tan(u)$:

$$\begin{aligned}\int x^3 \tan^3(x^4) \sec^4(x^4) dx &= \frac{1}{4} \int \tan^3(u) \sec^4(u) du \\ &= \frac{1}{4} \int \tan^3(u) (1 + \tan^2(u)) \sec^2(u) du \\ &= \frac{1}{4} \int v^3 (1 + v^2) dv \\ &= \frac{1}{4} \left(\frac{v^4}{4} + \frac{v^6}{6} \right) + c \\ &= \frac{\tan^4(x^4)}{16} + \frac{\tan^6(x^4)}{24} + c\end{aligned}$$

Item c) Usamos a substituição trigonométrica $x + 2 = \tan(\theta)$:

$$\begin{aligned}\int \frac{x+3}{(x^2+4x+5)^2} dx &= \int \frac{x+3}{((x+2)^2+1)^2} dx \\ &= \int \frac{\tan(\theta)+1}{(\tan^2(\theta)+1)^2} \sec^2(\theta) d\theta \\ &= \int \frac{\tan(\theta)+1}{\sec^2(\theta)} d\theta \\ &= \int \sin(\theta) \cos(\theta) d\theta + \int \cos^2(\theta) d\theta \\ &= \frac{\sin^2(\theta)}{2} + \frac{1}{2}(\theta + \sin(\theta) \cos(\theta)) + c \\ &= \frac{(x+2)^2}{2(x^2+4x+5)} + \frac{1}{2} \left(\arctan(x+2) + \frac{x+2}{x^2+4x+5} \right) + c\end{aligned}$$