Questão 1 (Turma A)

Item a) Aplicamos duas vezes a integração por partes:

$$\begin{aligned}\int x^2(\ln(x))^2dx =& \frac{x^3}{3}(\ln(x))^2 - \frac{2}{3}\int x^2\ln(x)dx \\ =& \frac{x^3}{3}(\ln(x))^2 - \frac{2}{3}(\frac{x^3}{3}\ln(x) - \frac{1}{3}\int x^2dx) \\ =& \frac{x^3}{3}(\ln(x))^2 - \frac{2}{9}x^3\ln(x) + \frac{2}{27}x^3 + c\end{aligned}$$

Item b) Usamos a substituição $u = x^3$ e, em seguida, $v = \tan(u)$:

$$\begin{aligned}\int x^2\tan^3(x^3)\sec^4(x^3)dx =& \frac{1}{3}\int \tan^3(u)\sec^4(u)du \\ =& \frac{1}{3}\int \tan^3(u)(1+\tan^2(u))\sec^2(u)du \\ =& \frac{1}{3}\int v^3(1+v^2)dv \\ =& \frac{1}{3}\left(\frac{v^4}{4} + \frac{v^6}{6}\right) + c \\ =& \frac{\tan^4(x^3)}{12} + \frac{\tan^6(x^3)}{18} + c\end{aligned}$$

Item c) Usamos a substituição trigonométrica $x - 2 = \tan(\theta)$:

$$\begin{aligned}\int \frac{x+1}{(x^2-4x+5)^2}dx =& \int \frac{x+1}{((x-2)^2+1)^2}dx \\ =& \int \frac{\tan(\theta)+3}{(\tan^2(\theta)+1)^2}\sec^2(\theta)d\theta \\ =& \int \frac{\tan(\theta)+3}{\sec^2(\theta)}d\theta \\ =& \int \operatorname{sen}(\theta)\cos(\theta)d\theta + 3\int \cos^2(\theta)d\theta \\ =& \frac{\operatorname{sen}^2(\theta)}{2} + \frac{3}{2}(\theta + \operatorname{sen}(\theta)\cos(\theta)) + c \\ =& \frac{(x-2)^2}{2(x^2-4x+5)} + \frac{3}{2}\left(\arctan(x-2) + \frac{x-2}{x^2-4x+5}\right) + c\end{aligned}$$