Questão 3 - A

Precisamos determinar a intersecção entre as funções $f(x) = \left(1+\frac{2}{x}\right)e^{\frac{2}{x}}$ $(x \neq 0)$ e $g(x) = k$. Inicialmente esbocemos o gráfico da f.

$$f'(x) = -\frac{2}{x^2}e^{\frac{2}{x}} - \frac{2}{x^2}\left(1+\frac{2}{x}\right)e^{\frac{2}{x}}$$

$$= -\frac{2}{x^2}e^{\frac{2}{x}}\left(2+\frac{2}{x}\right) = \frac{-4(x+1)\overbrace{e^{\frac{2}{x}}}^{>0}}{x^3}$$

		-1		0	
$-4(x+1)$	$+$		$-$		$-$
x^3	$-$		$-$	$\circ$	$+$
f'	$-$		$+$	$\circ$	$-$
	$\searrow$		$\nearrow$		$\searrow$

mínimo local

$f(-1) = -e^{-2}$

$$\lim_{x\to 0^+} \underbrace{\left(1+\frac{2}{x}\right)}_{\to +\infty} \underbrace{e^{\frac{2}{x}}}_{\to +\infty} = +\infty$$

$$\lim_{x\to 0^-} \underbrace{\left(1+\frac{2}{x}\right)}_{\to -\infty} \underbrace{e^{\frac{2}{x}}}_{\to 0} =$$

$$= \lim_{x\to 0^-} \frac{1+\frac{2}{x} \to -\infty}{e^{-\frac{2}{x}} \to +\infty}$$

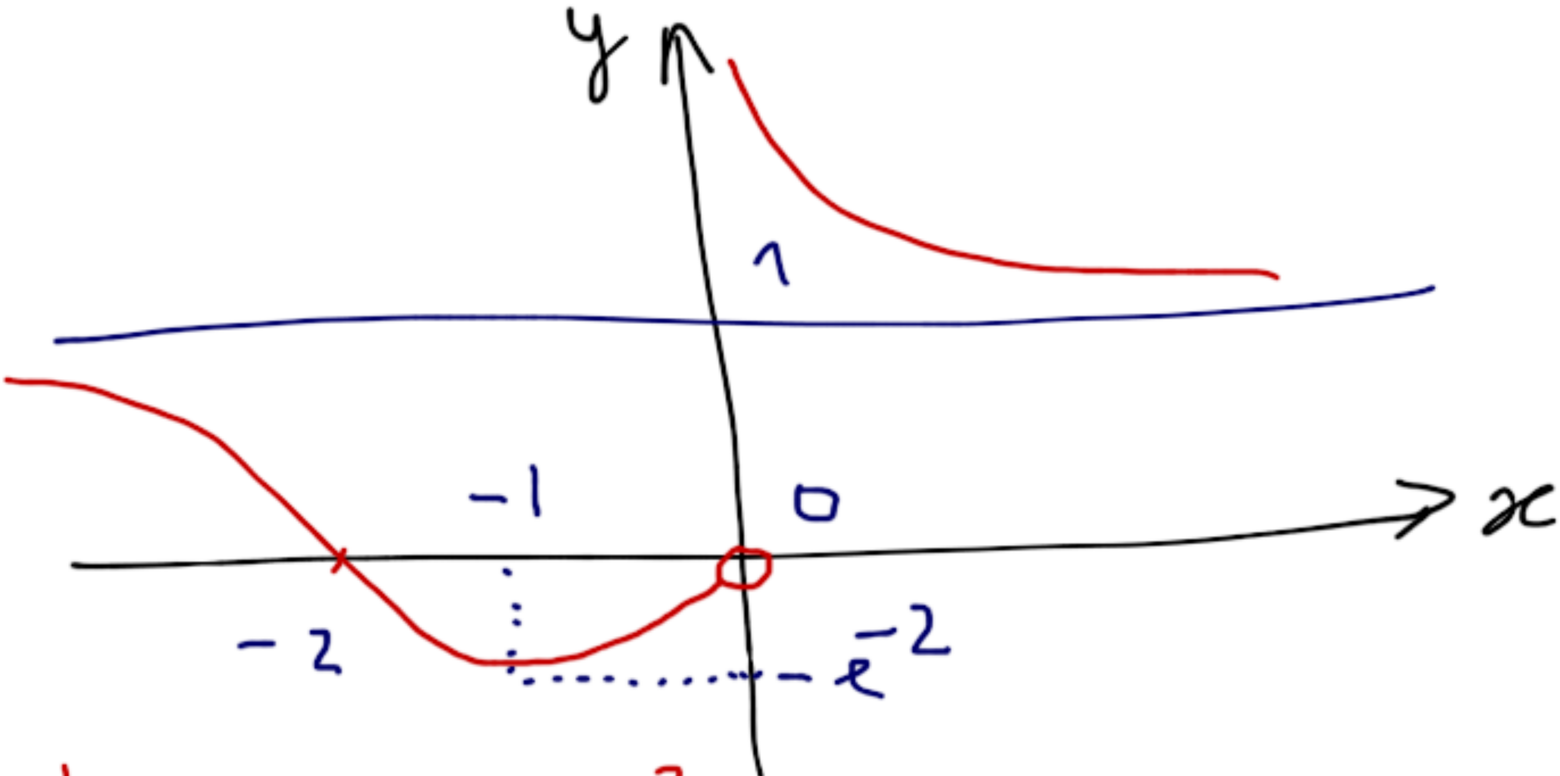
L'Hospital

$$= \lim_{x\to 0^-} \frac{-\frac{2}{x^2}}{\frac{2}{x^2} e^{-\frac{2}{x}}} =$$

$$= -\lim_{x\to 0^-} e^{\frac{2}{x}} = 0$$

$$\lim_{x\to +\infty} \left(1+\underbrace{\frac{2}{x}}_{\to 0}\right) \underbrace{e^{\frac{2}{x}}}_{\to 1} = 1$$

$$\lim_{x\to-\infty}\left(1+\underbrace{\frac{2}{x}}_{\to 0}\right)\underbrace{e^{\frac{2}{x}}}_{\to 1} = 1$$



$k=1$ ou $k<-e^{-2} \Longrightarrow$ zero soluções

$k=-e^{-2}$ ou $(k\geq 0$ e $k\neq 1) \Longrightarrow$ 1 solução

$-e^{-2}<k<0 \Longrightarrow$ 2 soluções