1. (3,0) Calcule os seguintes limites, caso existam. **Justifique!**

   (a) $\displaystyle\lim_{x\to+\infty} \frac{x^3\cos x + x^5 \operatorname{sen}(\frac{1}{x})}{3x^4+2x^2+5}$

   (b) $\displaystyle\lim_{x\to 3^-} \frac{\sqrt{10-x^2}-1}{\sqrt{x^3-6x^2+9x}}$

   (c) $\displaystyle\lim_{x\to-\infty} (\sqrt[3]{x^3-x^2}-x)$

(a) $\displaystyle\lim_{x\to+\infty} \frac{x^3\cos x + x^5 \operatorname{sen}\left(\frac{1}{x}\right)}{3x^4+2x^2+5} = \lim_{x\to+\infty} \frac{\overbrace{\frac{\cos x}{x}}^{\to 0} + \overbrace{x \operatorname{sen}\left(\frac{1}{x}\right)}^{\to 1}}{3+\underbrace{\frac{2}{x^2}}_{\to 0}+\underbrace{\frac{5}{x^4}}_{\to 0}} = \frac{1}{3}$

Justificativa

(i) $\displaystyle\lim_{x\to+\infty} \cos x \cdot \frac{1}{x} = 0$, pois $|\cos x| \leq 1$ para todo x e $\displaystyle\lim_{x\to+\infty} \frac{1}{x} = 0$

(ii) $\displaystyle\lim_{x\to+\infty} x \operatorname{sen}\left(\frac{1}{x}\right) = \lim_{x\to+\infty} \frac{\operatorname{sen}\left(\frac{1}{x}\right)}{\frac{1}{x}} = 1$ pois $\displaystyle\lim_{x\to+\infty} \frac{1}{x} = 0$ e $\displaystyle\lim_{t\to 0} \frac{\operatorname{sen} t}{t} = 1$

(b) $\displaystyle\lim_{x\to 3^-} \frac{\sqrt{10-x^2}-1}{\sqrt{x^3-6x^2+9x}} = \lim_{x\to 3^-} \frac{10-x^2-1}{(\sqrt{10-x^2}+1)\sqrt{x^3-6x^2+9x}}$

$\displaystyle= \lim_{x\to 3^-} \frac{(3+x)(3-x)}{(\sqrt{10-x^2}+1)(\sqrt{x(x-3)^2})} = \lim_{x\to 3^-} \frac{(3+x)(3-x)}{(\sqrt{10-x^2}+1)|x-3|\sqrt{x}} \overset{(*)}{=}$

$\displaystyle= \lim_{x\to 3^-} \frac{(3+x)}{(\sqrt{10-x^2}+1)\sqrt{x}} = \frac{6}{2\sqrt{3}} = \sqrt{3}$

(*) Para $x < 3$, temos $|x-3| = 3-x$

(c)

$$\left(\sqrt[3]{x^3-x^2}-x\right)\left[\left(\sqrt[3]{x^3-x^2}\right)^2+x\sqrt[3]{x^3-x^2}+x^2\right]=x^3-x^2-x^3=-x^2$$

$$\lim_{x\to-\infty}\left(\sqrt[3]{x^3-x^2}-x\right)=\lim_{x\to-\infty}\frac{-x^2}{\left(\sqrt[3]{x^3-x^2}\right)^2+x\sqrt[3]{x^3-x^2}+x^2}=$$

$$=\lim_{x\to-\infty}\frac{-x^2}{\left(x\sqrt[3]{1-\frac{1}{x}}\right)^2+xx\sqrt[3]{1-\frac{1}{x}}+x^2}=$$

$$=\lim_{x\to-\infty}\frac{-1}{\left(\sqrt[3]{1-\frac{1}{x}}\right)^2+\sqrt[3]{1-\frac{1}{x}}+1}=-\frac{1}{3}$$