1. (3,0) Calcule os seguintes limites, caso existam. **Justifique!**

(a) $\lim\limits_{x\to+\infty} \dfrac{x^3\cos x + x^5 \operatorname{sen}(\frac{1}{x})}{2x^4+5x^2+4}$

(b) $\lim\limits_{x\to 2^-} \dfrac{\sqrt{5-x^2}-1}{\sqrt{x^3-4x^2+4x}}$

(c) $\lim\limits_{x\to-\infty} (\sqrt[3]{x^3+x^2}-x)$

(a) $\lim\limits_{x\to+\infty} \dfrac{x^3\cos x + x^5\operatorname{sen}\left(\frac{1}{x}\right)}{2x^4+5x^2+4} = \lim\limits_{x\to+\infty} \dfrac{\overset{\to 0}{\dfrac{\cos x}{x}} + \overset{\to 1}{x\operatorname{sen}\left(\frac{1}{x}\right)}}{2+\underset{0}{\dfrac{5}{x^2}}+\underset{\to 0}{\dfrac{4}{x^4}}} = \dfrac{1}{2}$

Justificativa

(i) $\lim\limits_{x\to+\infty} \cos x \cdot \dfrac{1}{x} = 0$ pois $|\cos x| \leq 1$ para todo x

e $\lim\limits_{x\to+\infty} \dfrac{1}{x} = 0$

(ii) $\lim\limits_{x\to+\infty} x \operatorname{sen}\left(\dfrac{1}{x}\right) = \lim\limits_{x\to+\infty} \dfrac{\operatorname{sen}\left(\frac{1}{x}\right)}{\frac{1}{x}} = 1$ pois

$\lim\limits_{x\to+\infty} \dfrac{1}{x} = 0$ e $\lim\limits_{t\to 0} \dfrac{\operatorname{sen} t}{t} = 1$

(b) $\lim\limits_{x\to 2^-} \dfrac{\sqrt{5-x^2}-1}{\sqrt{x^3-4x^2+4x}} = \lim\limits_{x\to 2^-} \dfrac{5-x^2-1}{\left(\sqrt{5-x^2}+1\right)\sqrt{x^3-4x^2+4x}} =$

$= \lim\limits_{x\to 2^-} \dfrac{(2+x)(2-x)}{\left(\sqrt{5-x^2}+1\right)\sqrt{x(x-2)^2}} = \lim\limits_{x\to 2^-} \dfrac{(2+x)(2-x)}{\left(\sqrt{5-x^2}+1\right)|x-2|\sqrt{x}} \overset{(*)}{=}$

$= \lim\limits_{x\to 2^-} \dfrac{2+x}{\left(\sqrt{5-x^2}+1\right)\sqrt{x}} = \dfrac{4}{2\sqrt{2}} = \sqrt{2}$

(*) Para $x < 2$, temos $|x-2| = 2-x$

(c)

$$\left(\sqrt[3]{x^3+x^2}-x\right)\left[\left(\sqrt[3]{x^3+x^2}\right)^2+x\sqrt[3]{x^3+x^2}+x^2\right]=x^3+x^2-x^3=x^2$$

$$\lim_{x\to-\infty}\left(\sqrt[3]{x^3+x^2}-x\right)=\lim_{x\to-\infty}\frac{x^2}{\left(\sqrt[3]{x^3+x^2}\right)^2+x\sqrt[3]{x^3+x^2}+x^2}=$$

$$\lim_{x\to-\infty}\frac{x^2}{\left(x\sqrt[3]{1+\frac{1}{x}}\right)^2+x\cdot x\sqrt[3]{1+\frac{1}{x}}+x^2}=$$

$$=\lim_{x\to-\infty}\frac{1}{\left(\sqrt[3]{1+\frac{1}{x}}\right)^2+\sqrt[3]{1+\frac{1}{x}}+1}=\frac{1}{3}$$