Questão 3.

I) Sejam $f(x) = \sqrt[5]{x}$ e $x_0 \neq 0$.

(1,0) a) Determine o polinômio de Taylor de ordem 3 de f em volta de x_0.

$$p_3(x) = x_0^{\frac{1}{5}} + \tfrac{1}{5}x_0^{-\frac{4}{5}}(x - x_0) - \tfrac{4}{5^2.2}x_0^{-\frac{9}{5}}(x - x_0)^2 + \tfrac{36}{5^3.3!}x_0^{-\frac{14}{5}}(x - x_0)^3$$

$$E(x) = \tfrac{f^{(4)}(\bar{x})}{4!}(x - x_0)^4 = -\tfrac{504}{5^4.4!}\bar{x}^{-\frac{19}{5}}(x - x_0)^4 = -\tfrac{21}{5^4}\bar{x}^{-\frac{19}{5}}(x - x_0)^4$$

(1,0) b) Usando o item (a) para $x_0 = 32$, encontre um valor aproximado para $\sqrt[5]{34}$ e decida se o erro, em módulo, é inferior a $\dfrac{1}{5^2 \cdot 2^{15}}$.

$$p_3(x) = 2 + \tfrac{1}{5.2^4}(x - 32) - \tfrac{1}{5^2.2^8}(x - 32)^2 + \tfrac{3}{5^3.2^{13}}(x - 32)^3$$

$$\sqrt[5]{34} \approx p_3(34) = 2 + \tfrac{1}{5.2^4}(34 - 32) - \tfrac{1}{5^2.2^8}(34 - 32)^2 + \tfrac{3}{5^3.2^{13}}(34 - 32)^3$$

$$\sqrt[5]{34} \approx 2 + \tfrac{1}{40} - \tfrac{1}{1600} + \tfrac{3}{128000} = 2 + 0,025 - 0,000625 + 0,0000234 = 2,0243984.$$

$$E(x) = -\tfrac{21}{5^4\bar{x}^{\frac{19}{5}}} \cdot (x - 32)^4 \Rightarrow E(34) = -\tfrac{21}{5^4\bar{x}^{\frac{19}{5}}} \cdot (34 - 32)^4 = -\tfrac{21}{25.5^2\bar{x}^{\frac{19}{5}}} \cdot 2^4$$

Como $\bar{x} > 32$, pois $32 < \bar{x} < 34$, temos que $\frac{1}{\bar{x}} < \frac{1}{32} = \frac{1}{2^5}$.

Portanto, $|E(34)| = |-\tfrac{21}{25.5^2\bar{x}^{\frac{19}{5}}} \cdot 2^4| = \tfrac{21}{25} \cdot \tfrac{1}{5^2} \cdot \tfrac{1}{\bar{x}^{\frac{19}{5}}} \cdot 2^4 < \tfrac{21}{25} \cdot \tfrac{1}{5^2} \cdot \tfrac{2^4}{2^{19}} < \tfrac{1}{5^2.2^{15}}.$

(1,5) II) Seja $F(x) = \displaystyle\int_{\cos x}^{\operatorname{sen} x} e^{4x+t^2}\,dt$. Calcule $F'(x)$ e determine $F'(\frac{\pi}{4})$.

$$F'(x) = 4e^{4x}.\int_{\cos x}^{\operatorname{sen} x} e^{t^2}\,dt + e^{4x}[e^{\operatorname{sen}^2 x}.\cos x - e^{\cos^2 x}.(-\operatorname{sen} x)]$$

$$F'(\tfrac{\pi}{4}) = 4.0 + e^{\pi}\left[e^{\frac{1}{2}}.\frac{\sqrt{2}}{2} - e^{\frac{1}{2}}.(-\frac{\sqrt{2}}{2})\right] = e^{\pi+\frac{1}{2}}\sqrt{2}$$