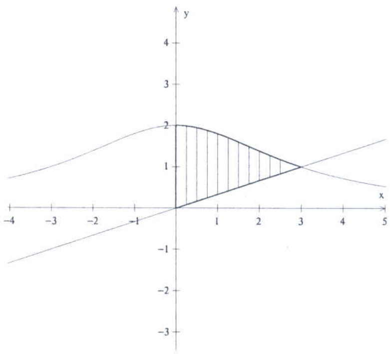
Questão 2. Seja R a região compreendida entre os gráficos de $f(x) = \dfrac{18}{9+x^2}$ e $g(x) = \dfrac{x}{3}$ para $x \in [0,3]$, como mostra a figura abaixo:



(2,0) a) Calcule o volume do sólido obtido pela rotação de R em torno do eixo Ox.

(1,5) b) Calcule o volume do sólido obtido pela rotação de R em torno do eixo Oy.

a) $V_x = V_1 - V_2$

$$V_1 = \int_0^3 \pi \left(\frac{18}{9+x^2}\right)^2 dx = 18^2 \pi \int_0^3 \frac{1}{(9+x^2)^2}\, dx \underset{x = 3\,tg\,t}{=}$$

$$= 18^2 \pi \int_0^{\pi/4} \frac{3 \sec^2 t}{(9+9\,tg^2 t)^2}\, dt = 12\pi \int_0^{\pi/4} \frac{1}{\sec^2 t}\, dt =$$

$$= 12\pi \int_0^{\pi/4} \cos^2 t \, dt = 12\pi \int_0^{\pi/4} \left(\frac{1}{2} + \frac{1}{2}\cos(2t)\right) dt =$$

$$= 12\pi \left[\frac{1}{2}\frac{\pi}{4} + \frac{1}{2}\frac{\operatorname{sen}(2t)}{2} \Big|_0^{\pi/4} \right] = 12\pi\left(\frac{\pi}{8} + \frac{1}{4}\right)$$

$$= \frac{3}{2}\pi^2 + 3\pi$$

$$V_2 = \int_0^3 \pi \left(\frac{x}{3}\right)^2 dx = \frac{\pi}{9}\frac{x^3}{3}\Big|_0^3 = \pi$$

$$V_x = \frac{3}{2}\pi^2 + 3\pi - \pi = \frac{3}{2}\pi^2 + 2\pi$$

A

b) $y = \frac{x}{3} \iff x = 3y$

$y = \frac{18}{9+x^2}, x \geq 0 \iff 9 + x^2 = \frac{18}{y} \quad x \geq 0 \iff$

$x = \sqrt{\frac{18}{y} - 9}$

$V_y = V_3 + V_4$

$V_3 = \int_0^1 \pi (3y)^2 dy = 9\pi \frac{y^3}{3} \Big|_0^1 = 3\pi$

$V_4 = \int_1^2 \pi \left(\sqrt{\frac{18}{y} - 9} \right)^2 dy = \pi \int_1^2 \left(\frac{18}{y} - 9 \right) dy =$

$= \pi \left(18 \ln y \Big|_1^2 - 9y \Big|_1^2 \right) = \pi \left(18 \ln 2 - 9 \right)$

$V_y = 3\pi + \pi(18 \ln 2 - 9) = \pi (18 \ln 2 - 6) = 6\pi(3\ln 2 - 1$