Questão 1. Calcule o limite ou justifique porque não existe:

a) (1,0 ponto) $\lim_{x\to 3} \dfrac{\sqrt[3]{x}-\sqrt[3]{3}}{x^2-6x+9}$

b) (1,0 ponto) $\lim_{x\to -\infty} \left(\sqrt{x^2+1}-\sqrt{x^2-5x+1}\right)$

c) (1,0 ponto) $\lim_{x\to 2} \dfrac{\operatorname{tg}^2(x-2)\operatorname{sen}\left(\frac{1}{x-2}\right)}{x-2}$

(a) $\lim_{x\to 3} \dfrac{\sqrt[3]{x}-\sqrt[3]{3}}{x^2-6x+9} = \lim_{x\to 3} \dfrac{\cancel{x-3}}{(x-3)^{\cancel{2}}\left(\sqrt[3]{x^2}+\sqrt[3]{3x}+\sqrt[3]{9}\right)} =$

$= \lim_{x\to 3} \dfrac{1}{\underbrace{(x-3)}_{0}\underbrace{\left(\sqrt[3]{x^2}+\sqrt[3]{3x}+\sqrt[3]{9}\right)}_{>0}}$

$\lim_{x\to 3^+} \dfrac{1}{\underbrace{(x-3)}_{\underset{0}{\to} >0}\underbrace{\left(\sqrt[3]{x^2}+\sqrt[3]{3x}+\sqrt[3]{9}\right)}_{>0}} = +\infty$ e $\lim_{x\to 3^-} \dfrac{1}{\underbrace{(x-3)}_{\underset{0}{\to} <0}\underbrace{\left(\sqrt[3]{x^2}+\sqrt[3]{3x}+\sqrt[3]{9}\right)}_{>0}} = -\infty$

Logo o limite não existe.

(b) $\lim_{x\to -\infty} \left(\sqrt{x^2+1}-\sqrt{x^2-5x+1}\right) = \lim_{x\to -\infty} \dfrac{\cancel{x^2}+\cancel{1}-\cancel{x^2}+5x-\cancel{1}}{\sqrt{x^2+1}+\sqrt{x^2-5x+1}} =$

$= \lim_{x\to -\infty} \dfrac{5x}{\underbrace{|x|}_{-x \text{ pois } x<0}\left(\sqrt{1+\frac{1}{x^2}}+\sqrt{1-\frac{5}{x}+\frac{1}{x^2}}\right)} = \lim_{x\to -\infty} \dfrac{5\cancel{x}}{-\cancel{x}\left(\sqrt{1+\underbrace{\frac{1}{x^2}}_{0}}+\sqrt{1-\underbrace{\frac{5}{x}}_{0}+\underbrace{\frac{1}{x^2}}_{0}}\right)} = -\dfrac{5}{2}$

c) $\lim_{x\to 2} \dfrac{\operatorname{tg}^2(x-2)\operatorname{sen}\left(\frac{1}{x-2}\right)}{x-2} = \overbrace{\lim_{x\to 2} \dfrac{\operatorname{sen}(x-2)}{\cos^2(x-2)} \cdot \dfrac{\operatorname{sen}(x-2)}{x-2} \cdot \operatorname{sen}\left(\dfrac{1}{x-2}\right)}^{\lim} = 0$

pois $\operatorname{sen}\left(\frac{1}{x-2}\right)$ é uma função limitada e

$\lim_{x\to 2} \dfrac{\overset{\to 0}{\operatorname{sen}(x-2)}}{\underset{\to 1}{\cos^2(x-2)}} \cdot \overset{\to 1}{\dfrac{\operatorname{sen}(x-2)}{x-2}} = 0$

<u>Obs.</u>: $\lim_{x\to 2} \dfrac{\operatorname{sen}(x-2)}{x-2} = \lim_{u\to 0} \dfrac{\operatorname{sen} u}{u} = 1$

$u = x-2 \qquad x\to 2 \Rightarrow u\to 0$