Questão 1. Calcule o limite ou justifique porque não existe:

a) (1,0 ponto) $\lim_{x \to 2} \frac{\sqrt[3]{x} - \sqrt[3]{2}}{x^2 - 4x + 4}$

b) (1,0 ponto) $\lim_{x \to -\infty} \left(\sqrt{x^2+1} - \sqrt{x^2 - 3x + 1}\right)$

c) (1,0 ponto) $\lim_{x \to 1} \frac{\operatorname{tg}^2(x-1)\operatorname{sen}\left(\frac{1}{x-1}\right)}{x-1}$

(a) $\lim_{x \to 2} \frac{\sqrt[3]{x} - \sqrt[3]{2}}{x^2 - 4x + 4} = \lim_{x \to 2} \frac{x-2}{(x-2)^2\left(\sqrt[3]{x^2} + \sqrt[3]{2x} + \sqrt[3]{4}\right)}$

$= \lim_{x \to 2} \frac{1}{\underbrace{(x-2)}_{0}\underbrace{\left(\sqrt[3]{x^2} + \sqrt[3]{2x} + \sqrt[3]{4}\right)}_{>0}}$

$\lim_{x \to 2^+} \frac{1}{\underbrace{(x-2)}_{\substack{0\\>0}}\underbrace{\left(\sqrt[3]{x^2} + \sqrt[3]{2x} + \sqrt[3]{4}\right)}_{>0}} = +\infty$, $\lim_{x \to 2^-} \frac{1}{\underbrace{(x-2)}_{\substack{0\\<0}}\underbrace{\left(\sqrt[3]{x^2} + \sqrt[3]{2x} + \sqrt[3]{4}\right)}_{>0}} = -\infty$

Logo o limite não existe

(b) $\lim_{x \to -\infty} \left(\sqrt{x^2+1} - \sqrt{x^2-3x+1}\right) = \lim_{x \to -\infty} \frac{\cancel{x^2} + \cancel{1} - \cancel{x^2} + 3x - \cancel{1}}{\sqrt{x^2+1} + \sqrt{x^2-3x+1}} =$

$= \lim_{x \to -\infty} \frac{3x}{\underbrace{|x|}_{\substack{\shortparallel \\ -x \text{ pois } x<0}}\left(\sqrt{1+\frac{1}{x^2}} + \sqrt{1 - \frac{3}{x} + \frac{1}{x^2}}\right)} = \lim_{x \to -\infty} \frac{3\cancel{x}}{-\cancel{x}\left(\sqrt{1+\underbrace{\frac{1}{x^2}}_{0}} + \sqrt{1 - \underbrace{\frac{3}{x}}_{0} + \underbrace{\frac{1}{x^2}}_{0}}\right)} = -\frac{3}{2}$

(c) $\lim_{x \to 1} \frac{\operatorname{tg}^2(x-1)\operatorname{sen}\left(\frac{1}{x-1}\right)}{x-1} = \overbrace{\lim_{x \to 1} \frac{\operatorname{sen}(x-1)}{\cos^2(x-1)} \cdot \frac{\operatorname{sen}(x-1)}{x-1}}^{\lim} \cdot \operatorname{sen}\left(\frac{1}{x-1}\right) = 0$

pois $\operatorname{sen}\left(\frac{1}{x-1}\right)$ é uma função limitada e

$\lim_{x \to 1} \frac{\overbrace{\operatorname{sen}(x-1)}^{\to 0}}{\underbrace{\cos^2(x-1)}_{\to 1}} \cdot \overbrace{\frac{\operatorname{sen}(x-1)}{x-1}}^{\to 1} = 0$

Obs: $\lim_{x \to 1} \frac{\operatorname{sen}(x-1)}{x-1} = \lim_{u \to 0} \frac{\operatorname{sen} u}{u} = 1$

$u = x - 1 \qquad x \to 1 \Rightarrow u \to 0$