

Questão 4. (2,5) Calcule:

B

(a) $\int_{\frac{5}{2}}^5 \frac{\sqrt{25-x^2}}{x^2} dx$

(b) $\int \ln(x^2+2x+2) dx$

a) $x = 5 \cos t, t \in [0, \pi]$

$dx = -5 \sin t dt$

$x = 5 \Rightarrow \cos t = 1 \Rightarrow t = 0$

$x = 5/2 \Rightarrow \cos t = 1/2 \Rightarrow t = \pi/3$

$$\int_{5/2}^5 \frac{\sqrt{25-x^2}}{x^2} dx = - \int_{\pi/3}^0 \frac{5 \sin^2 t}{\cos^2 t} dt = \int_0^{\pi/3} 5 \tan^2 t dt =$$

$$\int_0^{\pi/3} (5 \sec^2 t - 5) dt = 5 \left[\tan t \right]_0^{\pi/3} - 5 \left[t \right]_0^{\pi/3} = \boxed{\sqrt{3} - \pi/3}$$

b) $\int 1 \cdot \ln(x^2+2x+2) dx = x \ln(x^2+2x+2) - \int \frac{2x^2+2x}{x^2+2x+2} dx (*)$

\downarrow \downarrow
 f' g

$f'(x) = 1 \Rightarrow f(x) = x$

$g(x) = \ln(x^2+2x+2) \Rightarrow g'(x) = \frac{2x+2}{x^2+2x+2}$

$$\left| \frac{2x^2+2x}{x^2+2x+2} = 2 - \frac{2x+4}{x^2+2x+2} = 2 - \frac{(2x+2)}{x^2+2x+2} - \frac{2}{(x+1)^2+1} \right|$$

Logo,

$$\int \frac{2x^2+2x}{x^2+2x+2} dx = \int 2 dx - \int \frac{2x+2}{x^2+2x+2} dx - \int \frac{2}{1+(x+1)^2} dx$$

$$= 2x - \ln(x^2+2x+2) - 2 \arctg(x+1) + R, R \in \mathbb{R}$$

Daí, (*) = $\boxed{x \ln(x^2+2x+2) - 2x + \ln(x^2+2x+2) + 2 \arctg(x+1) + R, R \in \mathbb{R}}$