Questão 4. (2,5) Calcule: A

(a) $\displaystyle\int_{\frac{3}{2}}^{3} \frac{\sqrt{9-x^2}}{x^2}dx$ (b) $\displaystyle\int \ln(x^2-2x+2)dx$

a) $x = 3\cos t \Rightarrow dx = -3\,\text{sen}\,t\,dt$

$t \in [0, \pi]$

$\sqrt{9-x^2} = \sqrt{9-9\cos^2 t} = 3|\text{sen}\,t| = 3\,\text{sen}\,t$

$x = 3/2 \Leftrightarrow \cos t = 1/2 \Leftrightarrow t = \pi/3$

$x = 3 \Leftrightarrow \cos t = 1 \Leftrightarrow t = 0$

$$\int_{3/2}^{3} \frac{\sqrt{9-x^2}}{x^2}dx = -\int_{\pi/3}^{0} \frac{9\,\text{sen}^2 t}{9\cos^2 t} = \int_0^{\pi/3} \text{tg}^2 t\,dt$$

$$= \int_0^{\pi/3} (\sec^2 t - 1)\,dt = \text{tg}\,t\Big]_0^{\pi/3} - t\Big]_0^{\pi/3} = \underline{\underline{\sqrt{3} - \pi/3}}$$

b) $\displaystyle\int \underset{\substack{\downarrow \\ f'}}{1} . \underset{\substack{\downarrow \\ g}}{\ln(x^2-2x+2)}\,dx = x\ln(x^2-2x+2) - \int \frac{2x^2-2x}{x^2-2x+2}dx \quad (*)$

$f'(x) = 1 \Rightarrow f(x) = x$

$g(x) = \ln(x^2-2x+2) \Rightarrow g'(x) = \dfrac{2x-2}{x^2-2x+2}$

$$\frac{2x^2-2x}{x^2-2x+2} = 2 + \frac{2x-4}{x^2-2x+2} = 2 + \frac{2x-2}{x^2-2x+2} - \frac{2}{1+(x-1)^2}$$

Logo, $\displaystyle\int \frac{2x^2-2x}{x^2-2x+2}dx = \int 2dx + \int \frac{2x-2}{x^2-2x+2}dx - \int \frac{2dx}{1+(x-1)^2} =$

$= 2x + \ln(x^2-2x+2) - 2\,\text{arctg}(x-1) + k, \; k \in \mathbb{R}.$

Daí, $(*) = \boxed{x\ln(x^2-2x+2) - 2x - \ln(x^2-2x+2) + 2\,\text{arctg}(x-1) + k, \; k \in \mathbb{R}}$