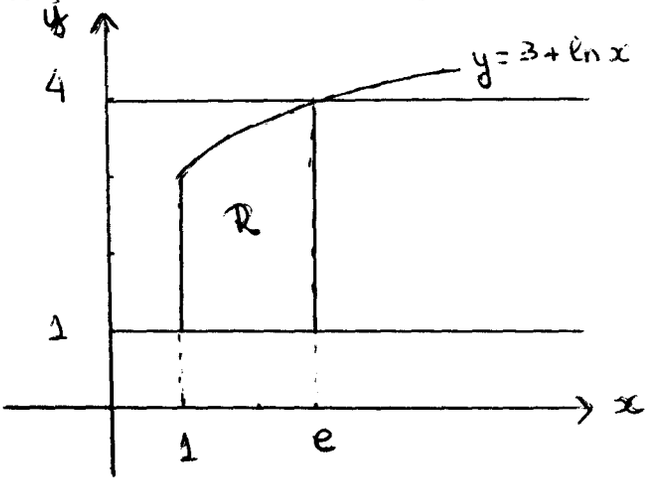
B

Questão 2. (2,5)

Seja $R = \{(x, y) \in \mathbb{R}^2 : 1 \leq x \leq e,\ 1 \leq y \leq 3 + \ln(x)\}$. Calcule o volume do sólido obtido pela rotação de R em torno da reta $y = 4$.



$$V = \pi \int_1^e \left[(4-1)^2 - (4-3-\ln x)^2\right] dx = \pi \int_1^e \left[8 + 2\ln x - \ln^2 x\right] dx =$$

$$= \pi \left[8(e-1) + 2\int_1^e \ln x\, dx - \int_1^e \ln^2 x\, dx\right]$$

Cálculo das integrais (por partes)

$$\int_1^e 1 \ln x\, dx = x \ln x \Big|_1^e - \int_1^e x \frac{1}{x} dx = (x \ln x - x) \Big|_1^e =$$

$$= (e \ln e) - (1 \ln 1 - 1) = 1$$

$$\int_1^e 1 \ln^2 x\, dx = x \ln^2 x \Big|_1^e - \int_1^e x (2 \ln x) \frac{1}{x} dx = x \ln^2 x \Big|_1^e - 2 \int_1^e \ln x\, dx =$$

$$= (e \ln^2 e - 1 \ln^2 1) - 2 = e - 2$$

Resposta: $V = \pi\left[8(e-1) + 2 - (e-2)\right] = \pi(7e-4)$