Questão 1. (3,0) a) Calcule, caso existam, os seguintes limites:

i) $$\lim_{x\to1^+}\frac{\sqrt{x-1}}{\sqrt[3]{x^2+1}-\sqrt[3]{2}}=\lim_{x\to1^+}\frac{\sqrt{x-1}}{\left(\sqrt[3]{x^2+1}-\sqrt[3]{2}\right)}\frac{\left(\sqrt[3]{(x^2+1)^2}+\sqrt[3]{x^2+1}\sqrt[3]{2}+\sqrt[3]{2^2}\right)}{\left(\sqrt[3]{(x^2+1)^2}+\sqrt[3]{x^2+1}\sqrt[3]{2}+\sqrt[3]{2^2}\right)}$$

$$=\lim_{x\to1^+}\frac{\sqrt{x-1}\,\left(\sqrt[3]{(x^2+1)^2}+\sqrt[3]{x^2+1}\sqrt[3]{2}+\sqrt[3]{2^2}\right)}{x^2-1}=\lim_{x\to1^+}\frac{\sqrt{x-1}\,\left(\sqrt[3]{(x^2+1)^2}+\sqrt[3]{x^2+1}\sqrt[3]{2}+\sqrt[3]{2^2}\right)}{(x-1)(x+1)}$$

$$=\lim_{x\to1^+}\frac{\sqrt{x-1}\,\left(\sqrt[3]{(x^2+1)^2}+\sqrt[3]{x^2+1}\sqrt[3]{2}+\sqrt[3]{2^2}\right)}{\sqrt{x-1}\,\sqrt{x-1}\,(x+1)}=\lim_{x\to1^+}\frac{\sqrt[3]{(x^2+1)^2}+\sqrt[3]{x^2+1}\sqrt[3]{2}+\sqrt[3]{2^2}}{\sqrt{x-1}\,(x+1)}=+\infty,$$

pois $\displaystyle\lim_{x\to1^+}\frac{\sqrt[3]{(x^2+1)^2}+\sqrt[3]{x^2+1}\sqrt[3]{2}+\sqrt[3]{2^2}}{x+1}=\frac{3\sqrt[3]{4}}{2}$ e $\displaystyle\lim_{x\to1^+}\frac{1}{\sqrt{x-1}}=+\infty$.

ii) $$\lim_{x\to+\infty}\frac{x+\operatorname{sen}x+2\sqrt{x}}{3x+\cos x}=\lim_{x\to+\infty}\frac{x\left(1+\frac{1}{x}\operatorname{sen}x+\frac{2}{\sqrt{x}}\right)}{x\left(3+\frac{1}{x}\cos x\right)}=\lim_{x\to+\infty}\frac{1+\frac{1}{x}\operatorname{sen}x+\frac{2}{\sqrt{x}}}{3+\frac{1}{x}\cos x}=\frac{1}{3},$$

pois $\displaystyle\lim_{x\to+\infty}\frac{1}{x}=\lim_{x\to+\infty}\frac{2}{\sqrt{x}}=0$, $|\operatorname{sen}x|\le1$ e $|\cos x|\le1$.

b) Verifique se a função

$$f(x)=\begin{cases}\dfrac{\sqrt{x^2-4x+4}}{x^2-3x+2-2(x-2)\cos(x-2)}, & \text{se } x\neq2\\[2mm] -1, & \text{se } x=2\end{cases}$$

é contínua em $x=2$. Justifique.

$$\lim_{x\to2}f(x)=\lim_{x\to1}\frac{\sqrt{x^2-4x+4}}{x^2-3x+2-2(x-2)\cos(x-2)}=\lim_{x\to2}\frac{\sqrt{(x-2)^2}}{(x-2)(x-1)-2(x-2)\cos(x-2)}$$

$$=\lim_{x\to2}\frac{|x-2|}{(x-2)\left[(x-1)-2\cos(x-2)\right]}$$

$$\left.\begin{array}{l}\displaystyle\lim_{x\to2^+}\frac{(x-2)}{(x-2)\left[(x-1)-2\cos(x-2)\right]}=\lim_{x\to2^+}\frac{1}{(x-1)-2\cos(x-2)}=-1\\[4mm]\displaystyle\lim_{x\to2^-}\frac{-(x-2)}{(x-2)\left[(x-1)-2\cos(x-2)\right]}=\lim_{x\to2^-}\frac{-1}{(x-1)-2\cos(x-2)}=1\end{array}\right\}\Longrightarrow\nexists\lim_{x\to2}f(x)$$

$\Longrightarrow f$ não é contínua em $x=2$.