**Questão 1.** (3,0) a) Calcule, caso existam, os seguintes limites:

i) $$\lim_{x\to 2^+} \frac{\sqrt{x-2}}{\sqrt[3]{x^2+1}-\sqrt[3]{5}} = \lim_{x\to 2^+} \frac{\sqrt{x-2}}{\left(\sqrt[3]{x^2+1}-\sqrt[3]{5}\right)} \frac{\left(\sqrt[3]{(x^2+1)^2}+\sqrt[3]{x^2+1}\sqrt[3]{5}+\sqrt[3]{5^2}\right)}{\left(\sqrt[3]{(x^2+1)^2}+\sqrt[3]{x^2+1}\sqrt[3]{5}+\sqrt[3]{5^2}\right)}$$

$$= \lim_{x\to 2^+} \frac{\sqrt{x-2}\,\left(\sqrt[3]{(x^2+1)^2}+\sqrt[3]{x^2+1}\sqrt[3]{5}+\sqrt[3]{5^2}\right)}{x^2-4} = \lim_{x\to 2^+} \frac{\sqrt{x-2}\,\left(\sqrt[3]{(x^2+1)^2}+\sqrt[3]{x^2+1}\sqrt[3]{5}+\sqrt[3]{5^2}\right)}{(x-2)(x+2)}$$

$$= \lim_{x\to 2^+} \frac{\sqrt{x-2}\,(\sqrt[3]{(x^2+1)^2}+\sqrt[3]{x^2+1}\sqrt[3]{5}+\sqrt[3]{5^2})}{\sqrt{x-2}\,\sqrt{x-2}\,(x+2)} = \lim_{x\to 2^+} \frac{\sqrt[3]{(x^2+1)^2}+\sqrt[3]{x^2+1}\sqrt[3]{5}+\sqrt[3]{5^2}}{\sqrt{x-2}\,(x+2)} = +\infty,$$

pois $\displaystyle\lim_{x\to 2^+} \frac{\sqrt[3]{(x^2+1)^2}+\sqrt[3]{x^2+1}\sqrt[3]{5}+\sqrt[3]{5^2}}{x+2} = \frac{3\sqrt[3]{25}}{4}$ e $\displaystyle\lim_{x\to 2^+} \frac{1}{\sqrt{x-2}} = +\infty$

ii) $$\lim_{x\to+\infty} \frac{2x+\cos x+3\sqrt{x}}{x+\operatorname{sen} x} = \lim_{x\to+\infty} \frac{x\left(2+\frac{1}{x}\cos x+\frac{3}{\sqrt{x}}\right)}{x\left(1+\frac{1}{x}\operatorname{sen} x\right)} = \lim_{x\to+\infty} \frac{2+\frac{1}{x}\cos x+\frac{3}{\sqrt{x}}}{1+\frac{1}{x}\operatorname{sen} x} = \frac{2}{1} = 2,$$

pois $\displaystyle\lim_{x\to+\infty} \frac{1}{x} = \lim_{x\to+\infty} \frac{3}{\sqrt{x}} = 0$, $|\cos x| \leq 1$ e $|\operatorname{sen} x| \leq 1$.

b) Verifique se a função

$$f(x) = \begin{cases} \dfrac{\sqrt{x^2-2x+1}}{x^2-3x+2+2(x-1)\cos(x-1)}, & \text{se } x \neq 1 \\ 1, & \text{se } x = 1 \end{cases}$$

é contínua em $x = 1$. Justifique.

$$\lim_{x\to 1} f(x) = \lim_{x\to 1} \frac{\sqrt{x^2-2x+1}}{x^2-3x+2+2(x-1)\cos(x-1)} = \lim_{x\to 1} \frac{\sqrt{(x-1)^2}}{(x-1)(x-2)+2(x-1)\cos(x-1)}$$

$$= \lim_{x\to 1} \frac{|x-1|}{(x-1)\left[(x-2)+2\cos(x-1)\right]}$$

$$\left.\begin{array}{l} \displaystyle\lim_{x\to 1^+} \frac{(x-1)}{(x-1)\left[(x-2)+2\cos(x-1)\right]} = \lim_{x\to 1^+} \frac{1}{(x-2)+2\cos(x-1)} = 1 \\ \displaystyle\lim_{x\to 1^-} \frac{-(x-1)}{(x-1)\left[(x-2)+2\cos(x-1)\right]} = \lim_{x\to 1^-} \frac{-1}{(x-2)+2\cos(x-1)} = -1 \end{array}\right\} \Longrightarrow \nexists \lim_{x\to 1} f(x)$$

$\Longrightarrow f$ não é contínua em $x = 1$.