Random Walks on Hyperbolic Groups

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The large-time behavior of the return probabilities of a random walk X_n is controlled by the behavior of its Green's function

$$G_r(x,y) = \sum_{n=0}^{\infty} r^n P^x\{X_n = y\}$$

at the radius $r = R$ of convergence. For nearest neighbor random walks on virtually free groups (groups that contain free groups as finite-index subgroups) the the Green's function is algebraic, and the singularity at the radius of convergence is of square-root type. This implies that the return probabilities behave as follows for $n \to \infty$:

$$P^x\{X_{2n} = x\} \sim CR^{-2n}n^{-3/2}.$$

For other hyperbolic groups, however, the Green's function is likely not algebraic, and so entirely different techniques are needed for the analysis of its leading singularity. We show that for simple random walk on a surface group of large genus the singularity is still of square-root type. A number of interesting related results concerning the behavior of $G_R(x,y)$ as y approaches the geometric boundary of the group are obtained:

(1) $G_R(x,y)$ decays exponentially in distance(x,y).
(2) Ancona's inequalities persist at $r = R$.
(3) The Martin boundary for $R-$potentials coincides with the geometric boundary.