1 Introduction to Complex Dynamics I
Friday 16/11: 10:00 to 12:00 in room B-5
and 14:00 to 16:00 in room B-144

Sylvain Bonnot
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Abstract

This first part of the mini-course will give a broad introduction to the field of complex dynamics, focussing mainly on polynomial iteration, with emphasis on the so-called *quadratic family*

$$P_c : z \in \mathbb{C} \mapsto z^2 + c,$$

where $c \in \mathbb{C}$.

Topics:

- Introduction: Newton's method as a dynamical system
- Local dynamics:
    1. Attracting fixed points: linearization theorem.
    2. Super-attracting fixed points: Böttcher's theorem and consequences.
    3. Parabolic fixed points: Leau-Fatou flower theorem, construction of Fatou coordinates.

As a good reference we cite [2] and the appendix of [1].

References

[1] Hubbard, J.H *Teichmüller theory, vol. 2 Matrix Editions*.

[2] Milnor, J. *Dynamics in one complex variable Annals of Mathematics Studies* (2006).

2 Introduction to Complex Dynamics II
Saturday 17/11: 10:00 to 12:00 in Auditorium Jacy Monteiro and 14:00 to 16:00 in Auditorium Jacy Monteiro

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Abstract

The second part of this introduction introduces the main players of the area, the so-called **Julia sets**, and the **filled Julia sets**:

$$K_c := \{z \in \mathbb{C} | \text{orbit}(z) \text{ is bounded}\}.$$

Topics:

- Julia sets, Fatou sets, with basic properties.
- Models for Julia sets.
- Parameter spaces: the Mandelbrot set.

References

[1] Hubbard, J.H *Teichmüller theory, vol. 2 Matrix Editions*.

[2] Milnor, J. *Dynamics in one complex variable*, *Annals of Mathematics Studies* (2006).

3 Introduction to Complex Dynamics III
Monday 19/11: 10:00 to 12:00 in Auditorium Jacy Monteiro and 14:00 to 16:00 in Auditorium Jacy Monteiro

Sylvain Bonnot
IME-USP

Luna Lomonaco
IME-USP

Abstract

The morning session (S. Bonnot) will give an overview of the higher dimensional theory, focussing on the dynamics of polynomial endomorphisms of $\mathbb{C}^2$.

The afternoon session (L. Lomonaco) will be an introduction to quasiconformal techniques.

4 Parabolic implosion
Wednesday 21/11: 10:00 to 12:00 in Auditorium Jacy Monteiro

Luna Lomonaco
IME-USP

Abstract

At a parabolic point, the function $c \mapsto K_c$ undergoes a discontinuity called *parabolic implosion*. This talk will describe the phenomenon together with relevant techniques (Horn maps, Ecalle-Voronin invariants).

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5 No-wandering domain theorem
Wednesday 21/11: 14:00 to 16:00 in Auditorium Jacy Monteiro

John Hubbard
Cornell University

Abstract

This talk will describe one of the most spectacular application of the quasiconformal techniques, namely Sullivan's No Wandering domain theorem.

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6 On critical point for linear cocycles and application to Hénon maps
Wednesday 21/11: 16:00 to 17:00 in Auditorium Jacy Monteiro

Francisco Valenzuela–Henriquez
PUCV

Abstract

In this talk, we introduce the solely dynamical obstruction to dominated splitting, namely critical points, for two–dimensional linear cocycles. With this notion, we expose application and some question related whit complex Hénon maps.

7 Parabolic blow-ups
Thursday 22/11: 10:00 to 12:00 in Auditorium Jacy Monteiro

John Hubbard
Cornell University

Abstract

T.B.A.

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8 The scenery flow for hyperbolic Julia sets
Thursday 22/11: 14:00 to 15:00 in Auditorium Jacy Monteiro

Albert Fisher
IME-USP

Abstract

I will give an overview of work carried out with P. Arnoux, T. Bedford, M. Talet and M. Urbanski, emphasizing the case of hyperbolic Julia sets.

We study the flow defined by zooming toward a point of a geometric object embedded in an ambient Euclidean space. For a smooth submanifold this is rather boring, as in the limit one sees the tangent space at the point, but for irregular objects like fractal sets the scenery at small scales keeps changing. A mathematical challenge is to precisely define this "scenery flow" and study its dynamical properties. Consider the limit set of a Fuchsian or Kleinian group; as we shall sketch, the scenery flow is isomorphic to the geodesic flow of the associated hyperbolic surface or three-manifold (more precisely, to the frame flow in the latter case). Now Sullivan proved a formula for the Hausdorff dimension of the limit set, that it equals the geodesic flow entropy.

Separately, Ruelle extended Bowen's formula for Hausdorff dimension of these limit sets, given in terms of a zero of the pressure function for log of the derivative,

to hyperbolic Julia sets.

We explain how to construct the scenery flow for Julia sets, and why that unites Bowen's formula with Sullivans: indeed, in both cases, "Hausdorff dimension equals the entropy of the scenery flow".

For the Kleinian case, Brin and Gromov proved the frame flow is ergodic; we proved, with Bedford and Urbanski, that the Julia set scenery flow is ergodic, in all but exceptional cases.

The scenery flow thus provides an analogue for Julia sets of the geodesic flow.

Something similar works in a number of other situations, e.g. for hyperbolic Cantor sets, and for Cantor sets given by nested circle rotations of periodic type (recent work with M. Talet).

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9 Indeterminacy Loci of Iterate Maps in Moduli Space
Thursday 22/11: 15:00 to 16:00 in Auditorium Jacy Monteiro

Jan Kiwi
PUC-Chile

Abstract

We study the action of the iteration maps on moduli spaces of complex rational maps. The tools employed emerge from considering dynamical systems acting on the Berkovich projective line over an appropriate non-Archimedean field.

The moduli space rat_d of rational maps in one complex variable of degree $d \geq 2$ has a natural compactification by a projective variety $\overline{\mathrm{rat}}_d$ provided by geometric invariant theory. Given $n \geq 2$, the iteration map $\Phi_n : \mathrm{rat}_d \to \mathrm{rat}_{d^n}$, defined by $\Phi_n : [f] \mapsto [f^n]$, extends to a rational map $\Phi_n \overline{\mathrm{rat}}_d \dashrightarrow \overline{\mathrm{rat}}_{d^n}$. We characterize the elements of $\overline{\mathrm{rat}}_d$ which lie in the indeterminacy locus of Φ_n. This is a joint work with Hongming Nie (Hebrew University of Jerusalem).

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10 Renormalization of multicritical circle maps
Friday 23/11: 10:00 to 11:00 in Room B-5

Gabriela Estevez
ICMC

Abstract

In this talk, we present the renormalization operator for circle maps with irrational rotation number having a finite number of inflexible critical points. We show some asymptotic properties of this operator and give some conditions to get rigidity. A remarkable fact is that our result is valid for almost all irrational rotation number. Joint work with Pablo Guarino (UFF-Brazil).

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11 Filled Julia sets associated to Bratteli diagrams
Friday 23/11: 11:00 to 12:00 in Room B-5

Danilo Caprio
UNESP

Abstract

In this lecture, we define some Markov Chains associated to Vershik maps on Bratteli diagrams.

We study probabilistic and spectral properties of their transition opera- tors and we prove that the spectra of these operators are connected to filled

Julia sets of polynomial maps in higher dimensions. We also study topolog- ical properties of these spectra.

This is a joint work with Ali Messaoudi and Glauco Valle.

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12 Dynamics of Asymptotically Holomorphic Polynomial-like Maps
Friday 23/11: 14:00 to 15:00 in Room B-144

Edson de Faria
IME-USP

Abstract

In this talk we will discuss the dynamics of certain maps in the complex plane that arise naturally as deep renormalizations of asymptotically holomorphic extensions of C^r unimodal maps (with $r > 3$). The maps we investigate are infinitely renormalizable of bounded type. We establish C^2 a-priori bounds for such such maps. We also prove a version of the Fatou-Julia-Sullivan theorem and a topological straightening theorem in this setting. In particular, these maps do not have wandering domains and their Julia sets are locally connected. The talk is based on joint work with Trevor Clark and Sebastian van Strien.

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13 The dynamics of $n-$ circle inversions
Friday 23/11: 14:00 to 15:00 in Room B-144

Gerardo Honorato
CIMFAV-UV, Chile

Abstract

In this talk we will discuss dynamical aspects of a class of rational maps acting on $\overline{\mathbb{C}}$ which are singular perturbations of polynomials and which are called $n-$ circle inversion. In particular, we describe the dynamics in the Julia set as a quotient of a shift of n symbols, and we study the asymptotic behaviour of the measure of maximal entropy of those maps. This is a joint work with Rubén Hidalgo (UFRO, Chile) and Pancho Valenzuela-Henriquez (PUCV, Chile).