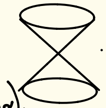
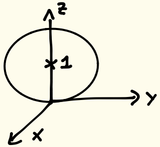
Lista 3 #4 (F): Calcule $\iiint_B dx\,dy\,dz$, onde B é limitado por $z^2 = x^2+y^2$ e $x^2+y^2+z^2 = 2z$.

<u>Determinação do domínio:</u> como eu expliquei na aula, $z^2 = x^2+y^2$ é a equação do cone .

Com as coordenadas esféricas: $z^2 = \rho^2\cos^2\alpha$, e $\underbrace{x^2+y^2}_{= r^2} = \rho^2\operatorname{sen}^2\alpha$ (lembra: $r = \rho\operatorname{sen}\alpha$).

Isto é, $\operatorname{sen}\alpha = \cos\alpha$, ou seja $\alpha = \frac{\pi}{4}$ ou $\alpha = \frac{3\pi}{4}$.

Agora: $x^2+y^2+z^2 = 2z \Leftrightarrow x^2+y^2+(z-1)^2 = 1$ equação da esfera de raio 1, centro $(0,0,1)$:



Com as coordenadas esféricas: $\underbrace{\rho^2}_{= x^2+y^2+z^2} = 2\underbrace{\rho\cos\alpha}_{= 2z}$, ou seja $\rho = 2\cos\alpha$.

<u>Esboço:</u>  ("cone de sorvete").

Podemos descrever B agora com coord. esféricas: $B = \{(\rho,\alpha,\theta) / 0 \leq \theta \leq 2\pi,\ 0 \leq \alpha \leq \frac{\pi}{4},\ 0 \leq \rho \leq 2\cos\alpha\}$.

<u>Cálculo da integral:</u>

$$\underbrace{V(B)}_{\text{volume}} = \int_{\theta=0}^{2\pi}\int_{\alpha=0}^{\pi/4}\int_{\rho=0}^{2\cos\alpha} \rho^2\operatorname{sen}\alpha\, d\rho\, d\alpha\, d\theta = \underbrace{\left(\int_{\theta=0}^{2\pi} d\theta\right)}_{= 2\pi}\cdot\left(\int_{\alpha=0}^{\pi/4} \operatorname{sen}\alpha \cdot \left[\frac{\rho^3}{3}\right]_0^{2\cos\alpha} d\alpha\right)$$

$$V(B) = \frac{2\pi}{3}\int_0^{\pi/4} 8\cos^3\alpha \cdot \operatorname{sen}\alpha\, d\alpha = \frac{16\pi}{3}\left[-\frac{\cos^4\alpha}{4}\right]_0^{\pi/4} = \frac{16\pi}{3}\left(-\frac{1}{4}\left(\left(\frac{\sqrt{2}}{2}\right)^4 - 1\right)\right)$$

$$= \frac{\cancel{16}\pi}{\cancel{3}}\,\frac{\cancel{3}}{\cancel{16}} = \pi.$$