Curvas paramétricas I

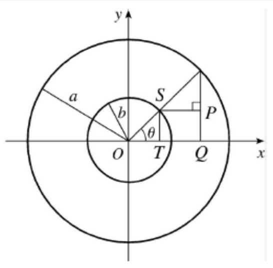
Exercício

Esboce as curvas com as seguintes equações paramétricas:

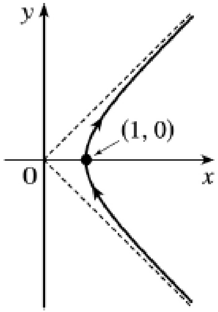
1. $x = sec(\theta)$ *e* $y = tg(\theta)$ *para* $\theta \in (-\pi/2, \pi/2)$.
2. $x = 1 + \cos\theta$ *e* $y = 2\cos\theta - 1$ *para* $\theta \in \mathbb{R}$

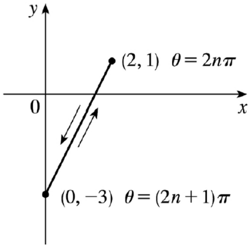
Exercício

Sejam a e b números fixos, descreve uma equação paramétrica (em θ*) para a curva feita de todas as posições possiveis do ponto P.*



Ex. 1:





Ex. 2: elipse $(x^2/a^2 + y^2/b^2 = 1)$.

Ex. 3:

Exercício

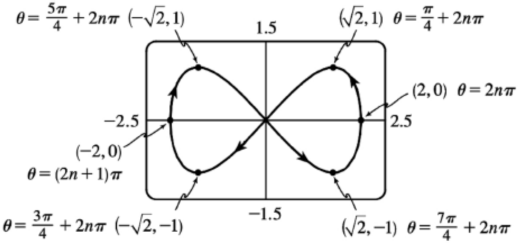
Encontre todos os pontos onde a reta tangente na curva $x = 2\cos\theta$, $y = \operatorname{sen}(2\theta)$ *é horizontal ou vertical. Esboce a curva.*

Ex. 4:

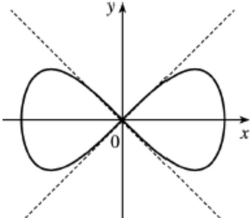
Exercício

Esboce a curva $x = \cos t$, $y = (\operatorname{sen} t).(\cos t)$. *Mostre que as retas* $y = x$ *e* $y = -x$ *são duas tangentes na curva, passando pela origem.*

Ex. 3:


$\theta = \frac{5\pi}{4} + 2n\pi$ $(-\sqrt{2}, 1)$
1.5
$(\sqrt{2}, 1)$ $\theta = \frac{\pi}{4} + 2n\pi$
$(2, 0)$ $\theta = 2n\pi$
-2.5
2.5
$(-2, 0)$
$\theta = (2n+1)\pi$
$\theta = \frac{3\pi}{4} + 2n\pi$ $(-\sqrt{2}, -1)$
-1.5
$(\sqrt{2}, -1)$ $\theta = \frac{7\pi}{4} + 2n\pi$

Ex. 4:


y
x
0

Ex. 5:

Exercício

Seja a curva $x = a(\cos\theta + \theta \mathrm{sen}\theta)$ *,* $y = a(\mathrm{sen}\theta - \theta\cos\theta)$ *com* $\theta \in [0, \pi]$*. Calcule o comprimento da curva.*

Ex. 6:

Exercício

Seja a curva $x = \frac{t}{1+t}$ *,* $y = \log(1+t)$ *com* $t \in [0, 2]$*. Calcule o comprimento da curva.(Pode utilizar symbolab.com no fim para calculara antiderivada).*

Ex. 5: Temos

$$\begin{aligned}(dx/d\theta)^2+(dy/d\theta)^2 &= a^2\left[(-\sin\theta+\theta\cos\theta+\sin\theta)^2+(\cos\theta+\theta\sin\theta-\cos\theta)^2\right]\\ &= a^2\theta^2\left(\cos^2\theta+\sin^2\theta\right)=(a\theta)^2\end{aligned}$$

então $L = (1/2)a\pi^2$.

Ex. 6:

$$.\ x=\frac{t}{1+t}\ ,\ y=\ln(1+t)\ ,\ 0\le t\le 2\ .\ \frac{dx}{dt}=\frac{(1+t)\cdot 1-t\cdot 1}{(1+t)^2}=\frac{1}{(1+t)^2}\qquad \frac{dy}{dt}=\frac{1}{1+t}\ ,$$

$$\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2=\frac{1}{(1+t)^4}+\frac{1}{(1+t)^2}=\frac{1}{(1+t)^4}\left[1+(1+t)^2\right]=\frac{t^2+2t+2}{(1+t)^4}\ .\ \text{Então,}$$

$$L=\int_0^2\frac{\sqrt{t^2+2t+2}}{(1+t)^2}\,dt=\int_1^3\frac{\sqrt{u^2+1}}{u^2}\,du[u=t+1,du=dt]=\left[-\frac{\sqrt{u^2+1}}{u}+\ln\left(u+\sqrt{u^2+1}\right)\right]_1^3$$

$$=-\frac{\sqrt{10}}{3}+\ln\left(3+\sqrt{10}\right)+\sqrt{2}-\ln\left(1+\sqrt{2}\right)$$

Ex. 7:

Exercício

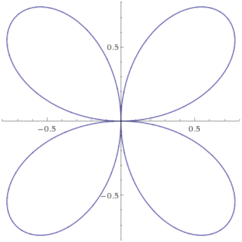
Escreve a curva $(x^2 + y^2)^3 = 4x^2y^2$ *na forma polar, e esboce a curva.*

Ex. 8:

Exercício

Mostre que a curva $r = a\mathrm{sen}\theta + b\cos\theta$ *é um circulo.*

Ex. 7: Temos



Ex. 8:

67. $r = a\sin\theta + b\cos\theta \Rightarrow r^2 = ar\sin\theta + br\cos\theta \Rightarrow x^2 + y^2 = ay + bx \Rightarrow$

$$x^2 - bx + \left(\frac{1}{2}b\right)^2 + y^2 - ay + \left(\frac{1}{2}a\right)^2 = \left(\frac{1}{2}b\right)^2 + \left(\frac{1}{2}a\right)^2 \Rightarrow \left(x - \frac{1}{2}b\right)^2 + \left(y - \frac{1}{2}a\right)^2 = \frac{1}{4}\left(a^2 + b^2\right)$$

Ex. 9:

Exercício

Esboce a região dentro da curva $r^2 = 8\cos 2\theta$ *e fora do disco* $r < 2$. *Calcule a área da região.*

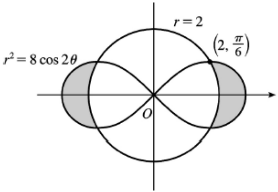
Ex. 10:

Exercício

Seja $F_1 = (-c, 0)$ *e* $F_2 = (c, 0)$ *e* $a > 0$. *Descreve o conjunto de todos os pontos* P *do plano tais que*

$$PF_1 - PF_2 = \pm 2a$$

Ex. 9: Temos



$$A = 4\int_0^{\pi/6}\left[\frac{1}{2}(8\cos 2\theta) - \frac{1}{2}(2)^2\right]d\theta = 8\int_0^{\pi/6}(2\cos 2\theta - 1)\,d\theta$$

$$= 8\left[\sin 2\theta - \theta\right]_0^{\pi/6} = 8\left(\sqrt{3}/2 - \pi/6\right) = 4\sqrt{3} - 4\pi/3$$

Ex. 10:

$$\left|PF_1\right| - \left|PF_2\right| = \pm 2a \Leftrightarrow \sqrt{(x+c)^2+y^2} - \sqrt{(x-c)^2+y^2} = \pm 2a \Leftrightarrow \sqrt{(x+c)^2+y^2} = \sqrt{(x-c)^2+y^2} \pm 2a \Leftrightarrow$$

$$(x+c)^2+y^2 = (x-c)^2+y^2+4a^2 \pm 4a\sqrt{(x-c)^2+y^2} \Leftrightarrow 4cx-4a^2 = \pm 4a\sqrt{(x-c)^2+y^2} \Leftrightarrow$$

$$c^2x^2-2a^2cx+a^4 = a^2\left(x^2-2cx+c^2+y^2\right) \Leftrightarrow \left(c^2-a^2\right)x^2-a^2y^2 = a^2\left(c^2-a^2\right) \Leftrightarrow b^2x^2-a^2y^2 = a^2b^2 \;(\text{ onde } b^2=c^2-a^2$$

$$) \Leftrightarrow \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$