

LISTA 1: REVISÃO DAS INTEGRAIS, PARTE I

Exercício 1. Calcule as seguintes integrais indefinidas:

1. $\int 5t^3 - 10t^{-6} + 4dt$ (Resp: $\frac{5}{4}t^4 + 2t^{-5} + 4t + C$)
2. $\int x^8 + x^{-8}dx$ (Resp: $\frac{1}{9}x^9 - \frac{1}{7}x^{-7} + C$)
3. $\int 3\sqrt[4]{x^3} + \frac{7}{x^5} + \frac{1}{6\sqrt{x}}dx$ (Resp: $\frac{12}{7}x^{7/4} - \frac{7}{4}x^{-4} + \frac{1}{3}x^{1/2} + C$)
4. $\int \frac{4x^{10} - 2x^4 + 15x^2}{x^3}dx$ (Resp: $\frac{1}{2}x^8 - x^2 + 15 \ln|x| + C$)
5. $\int 3e^x + 5\cos(x) - 10\sec^2x dx$ (resp: $3e^x + 5\sin x - 10 \operatorname{tg}(x) + C$)

Exercício 2. Determinar a única função f tal que: $f'(x) = 4x^3 - 9 + 2 \operatorname{sen} x + 7e^x$ e tal que $f(0) = 15$.

(Resp: $f(x) = x^4 - 9x - 2 \cos x + 7e^x + 10$).

Exercício 3. Determine as derivadas das seguintes funções:

- a) $g(x) = \int_{-4}^x e^{2t} \cos^2(1 - 5t) dt$ (Resp: $g'(x) = e^{2x} \cos^2(1 - 5x)$)
- b) $f(x) = \int_{x^2}^1 \frac{t^4 + 1}{t^2 + 1} dt$ (Resp: $f'(x) = -2x \cdot \frac{x^8 + 1}{x^4 + 1}$).

Exercício 4. Determine as integrais definidas:

- | | |
|---|---|
| 15. $\int_{\pi/2}^0 \frac{1 + \cos 2t}{2} dt$ | 16. $\int_{-\pi/3}^{\pi/3} \frac{1 - \cos 2t}{2} dt$ |
| 17. $\int_{-\pi/2}^{\pi/2} (8y^2 + \sin y) dy$ | 18. $\int_{-\pi/3}^{-\pi/4} \left(4 \sec^2 t + \frac{\pi}{t^2} \right) dt$ |
| 19. $\int_1^{-1} (r + 1)^2 dr$ | 20. $\int_{-\sqrt{3}}^{\sqrt{3}} (t + 1)(t^2 + 4) dt$ |
| 21. $\int_{\sqrt{2}}^1 \left(\frac{u^7}{2} - \frac{1}{u^5} \right) du$ | 22. $\int_{1/2}^1 \left(\frac{1}{v^3} - \frac{1}{v^4} \right) dv$ |
| 23. $\int_1^{\sqrt{2}} \frac{s^2 + \sqrt{s}}{s^2} ds$ | 24. $\int_9^4 \frac{1 - \sqrt{u}}{\sqrt{u}} du$ |
| 25. $\int_{-4}^4 x dx$ | 26. $\int_0^{\pi} \frac{1}{2} (\cos x + \cos x) dx$ |

Resp: 15) $-\pi/4$ 16) $\pi/3 - \sqrt{3}/4$ 17) $\frac{2}{3}\pi^3$ 18) $4\sqrt{3} - 3$ 19) $-8/3$
20) $10\sqrt{3}$ 21) $-3/4$ 22) $-5/6$ 23) $\sqrt{2} - \sqrt[4]{8} + 1$ 24) 3 25) 16 .

Exercício 5. Determine as integrais definidas (com a regra da substituição):

- | | |
|--|---|
| 1. a. $\int_0^3 \sqrt{y+1} dy$ | b. $\int_{-1}^0 \sqrt{y+1} dy$ |
| 2. a. $\int_0^1 r\sqrt{1-r^2} dr$ | b. $\int_{-1}^1 r\sqrt{1-r^2} dr$ |
| 3. a. $\int_0^{\pi/4} \tan x \sec^2 x dx$ | b. $\int_{-\pi/4}^0 \tan x \sec^2 x dx$ |
| 4. a. $\int_0^{\pi} 3 \cos^2 x \sin x dx$ | b. $\int_{2\pi}^{3\pi} 3 \cos^2 x \sin x dx$ |
| 5. a. $\int_0^1 t^3(1+t^4)^3 dt$ | b. $\int_{-1}^1 t^3(1+t^4)^3 dt$ |
| 6. a. $\int_0^{\sqrt{7}} t(t^2+1)^{1/3} dt$ | b. $\int_{-\sqrt{7}}^0 t(t^2+1)^{1/3} dt$ |
| 7. a. $\int_{-1}^1 \frac{5r}{(4+r^2)^2} dr$ | b. $\int_0^1 \frac{5r}{(4+r^2)^2} dr$ |
| 8. a. $\int_0^1 \frac{10\sqrt{v}}{(1+v^{3/2})^2} dv$ | b. $\int_1^4 \frac{10\sqrt{v}}{(1+v^{3/2})^2} dv$ |

Resp:

1. 14/3 e 2/3
2. 1/3 e 0
3. 1/2 e -1/2
4. 2 e 2
5. 15/16 e 0
6. 45/8 e -45/8
7. 0 e 1/8
8. 10/3 e 70/27.

Exercício 6. Determine as áreas das regiões sombreadas:

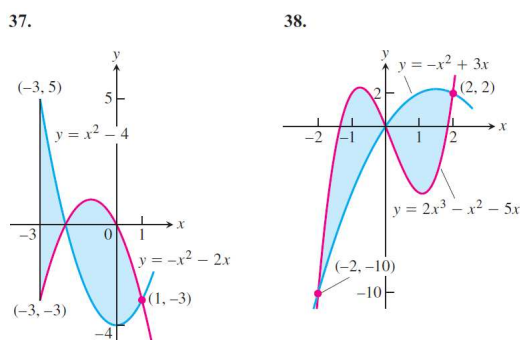


Figura 1.

Resp: 37) Área= 38/3 e 38) Área=16

Exercício 7. Calcule o valor medio das funções (no intervalo indicado):

1. $f(t) = t^2 - 5t + 6\cos(\pi t)$ em $[-1, 5/2]$ (Resp: $\frac{12}{7\pi} - \frac{13}{6}$)
2. $R(z) = \text{sen}(2z)e^{1-\cos(2z)}$ em $[-\pi, \pi]$ (Resp:=0).