LISTA 6: INTEGRAIS, PARTE I

Exercício 1. Calcule as seguintes integrais indefinidas:

1. $\int \ 5t^3 - 10t^{-6} + 4\,\mathrm{dt}$ (Resp:$\frac{5}{4}t^4 + 2t^{-5} + 4t + C$)

2. $\int \ x^8 + x^{-8}\mathrm{dx}$ (Resp: $\frac{1}{9}x^9 - \frac{1}{7}x^{-7} + C$)

3. $\int \ 3\sqrt[4]{x^3} + \frac{7}{x^5} + \frac{1}{6\sqrt{x}}\mathrm{dx}$ (Resp: $\frac{12}{7}x^{7/4} - \frac{7}{4}x^{-4} + \frac{1}{3}x^{1/2} + C$)

4. $\int \ \frac{4x^{10} - 2x^4 + 15x^2}{x^3}\mathrm{dx}$ (Resp:$\frac{1}{2}x^8 - x^2 + 15\ln|x| + C$)

5. $\int \ 3e^x + 5\cos(x) - 10\sec^2 x\,\mathrm{dx}$ (resp:$3e^x + 5\operatorname{sen} x - 10\operatorname{tg}(x) + C$)

Exercício 2. Determinar a única função f tal que: $f'(x) = 4x^3 - 9 + 2\operatorname{sen} x + 7e^x$ e tal que $f(0) = 15$.

(Resp: $f(x) = x^4 - 9x - 2\cos x + 7e^x + 10$).

Exercício 3. Determine as derivadas das seguintes funções:

a) $g(x) = \int_{-4}^{x} e^{2t}\cos^2(1 - 5t)\mathrm{dt}$ (Resp: $g'(x) = e^{2x}\cos^2(1 - 5x)$)

b) $f(x) = \int_{x^2}^{1} \frac{t^4 + 1}{t^2 + 1}\mathrm{dt}$ (Resp: $f'(x) = -2x.\frac{x^8 + 1}{x^4 + 1}$).

Exercício 4. Determine as integrais definidas:

15. $\int_{\pi/2}^{0} \frac{1 + \cos 2t}{2}\,dt$

16. $\int_{-\pi/3}^{\pi/3} \frac{1 - \cos 2t}{2}\,dt$

17. $\int_{-\pi/2}^{\pi/2} (8y^2 + \sin y)\,dy$

18. $\int_{-\pi/3}^{-\pi/4} \left(4\sec^2 t + \frac{\pi}{t^2}\right)dt$

19. $\int_{1}^{-1} (r + 1)^2\,dr$

20. $\int_{-\sqrt{3}}^{\sqrt{3}} (t + 1)(t^2 + 4)\,dt$

21. $\int_{\sqrt{2}}^{1} \left(\frac{u^7}{2} - \frac{1}{u^5}\right)du$

22. $\int_{1/2}^{1} \left(\frac{1}{v^3} - \frac{1}{v^4}\right)dv$

23. $\int_{1}^{\sqrt{2}} \frac{s^2 + \sqrt{s}}{s^2}\,ds$

24. $\int_{9}^{4} \frac{1 - \sqrt{u}}{\sqrt{u}}\,du$

25. $\int_{-4}^{4} |x|\,dx$

26. $\int_{0}^{\pi} \frac{1}{2}(\cos x + |\cos x|)\,dx$

Resp: 15) $-\pi/4$ 16) $\pi/3 - \sqrt{3}/4$ 17) $\frac{2}{3}\pi^3$ 18) $4\sqrt{3} - 3$ 19) -8/3

20) $10\sqrt{3}$ 21) -3/4   22) -5/6   23) $\sqrt{2} - \sqrt[4]{8} + 1$ 24) 3   25) 16.

Exercício 5. Determine as integrais definidas (com a regra da substituição):

1. a. $\displaystyle\int_0^3 \sqrt{y+1}\,dy$ b. $\displaystyle\int_{-1}^0 \sqrt{y+1}\,dy$

2. a. $\displaystyle\int_0^1 r\sqrt{1-r^2}\,dr$ b. $\displaystyle\int_{-1}^1 r\sqrt{1-r^2}\,dr$

3. a. $\displaystyle\int_0^{\pi/4} \tan x \sec^2 x\,dx$ b. $\displaystyle\int_{-\pi/4}^0 \tan x \sec^2 x\,dx$

4. a. $\displaystyle\int_0^{\pi} 3\cos^2 x \sin x\,dx$ b. $\displaystyle\int_{2\pi}^{3\pi} 3\cos^2 x \sin x\,dx$

5. a. $\displaystyle\int_0^1 t^3(1+t^4)^3\,dt$ b. $\displaystyle\int_{-1}^1 t^3(1+t^4)^3\,dt$

6. a. $\displaystyle\int_0^{\sqrt{7}} t(t^2+1)^{1/3}\,dt$ b. $\displaystyle\int_{-\sqrt{7}}^0 t(t^2+1)^{1/3}\,dt$

7. a. $\displaystyle\int_{-1}^1 \frac{5r}{(4+r^2)^2}\,dr$ b. $\displaystyle\int_0^1 \frac{5r}{(4+r^2)^2}\,dr$

8. a. $\displaystyle\int_0^1 \frac{10\sqrt{v}}{(1+v^{3/2})^2}\,dv$ b. $\displaystyle\int_1^4 \frac{10\sqrt{v}}{(1+v^{3/2})^2}\,dv$

Resp:

1. 14/3 e 2/3
2. 1/3 e 0
3. 1/2 e -1/2
4. 2 e 2
5. 15/16 e 0
6. 45/8 e -45/8
7. 0 e 1/8
8. 10/3 e 70/27.

**Exercício 6.** Determine as áreas das regiões sombreadas:
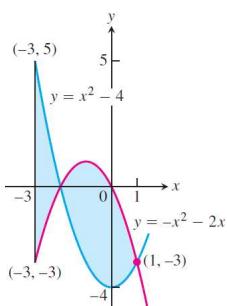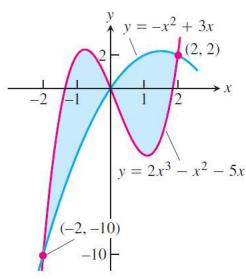
Figura 1.

Resp: 37) Área= 38/3   e 38) Área=16

Exercício 7. Calcule o valor medio das funções (no intervalo indicado):

1. $f(t)=t^2-5t+6\cos(\pi t)$ em $[-1,5/2]$ (Resp:$\frac{12}{7\pi}-\frac{13}{6}$)
2. $R(z)=\operatorname{sen}(2z)e^{1-\cos(2z)}$ em $[-\pi,\pi]$ (Resp:=0).