Abstracts presented to the Program Committee for the 13-th International Conference on Representations of Algebras, ICRA XIII, São Paulo 30 July–8 August, 2008

Contents

Abstracts presented for the Workshop in São Paulo, SP.

Koszul algebras, linear modules, and their generalizations

Edward Green
Virginia Tech
460 McBryde Hall
United States

E-mail address: **green@math.vt.edu**

Abstract: In the first two talks, I will present the definitions of Koszul algebras and linear modules. I will discuss Koszul duality and many homological properties of Koszul algebras and linear modules, while providing numerous examples. The last talk will be devoted to various generalizations of Koszul algebras; including definitions and results for D-Koszul algebras, quasi-Koszul algebras, and delta-Koszul algebras.

References

[] None

Representations of alternative and Jordan algebras

Ivan Shestakov
Universidade de Sao Paulo
Rua do Matao, 1010
Brazil

E-mail address: **shestak@ime.usp.br**

We will give an introduction to the structure theory and representations of finite dimensional alternative and Jordan algebras. In particular, the structure of simple algebras and of irreducible bimodules will be given, including the Morita - equivalence of the category of Jordan bimodules over a Jordan matrix algebra and the category of associative/alternative bimodules with involution over the corresponding coordinatizating composition algebra.

References

[] None

2-Calabi-Yau-tilted algebras

Idun Reiten
NTNU
Alfred Getz vei 1,
Norway

E-mail address: **idunr@math.ntnu.no**

Abstract: In this series we discuss some of the main properties of the 2-Calabi-Yau-tilted algebras. They are by definition the endomorphism algebras of cluster-tilting objects in Hom-finite triangulated 2-Calabi-Yau categories, and contain the cluster-tilted algebras.

References

[] None

Trivial extensions, iterated tilted algebras and cluster tilted algebras

María Inés Platzeck
Universidad Nacional del Sur
Av. Alem 1253
Argentina

E-mail address: **platzeck@uns.edu.ar**

Trivial extensions of artin algebras have been extensively studied and play and important role in the representation theory of artin algebras. For instance, there are interesting and useful connections with tilting theory. We will explore the connections between trivial extensions, iterated tilted algebras and cluster tilted algebras, and give some applications.

References

[] None

Fourier-Mukai transforms and the Hall algebra of an elliptic curve

Igor Burban
NULL
University of Bonn
Beringstr. 1
Germany

E-mail address: **burban@math.uni-bonn.de**

Lecture 1. Vector bundles on an elliptic curve via Fourier-Mukai transforms
Lecture 2. Hall algebra of an elliptic curve
Lecture 3. Finite length modules over the ring $\mathbf{k}[[x,y]]/xy$ and semi-stable sheaves on a nodal cubic curve

Abstract.
In my first lecture, I am going to explain Atiyah's classification of indecomposable vector bundles on an elliptic curve using the technique of twist functors and Fourier-Mukai transforms.
This description will be used in my second lecture (based on a joint paper with Olivier Schiffmann) to give a presentation of the composition subalgebra of the Hall algebra of an elliptic curve by generators and relations. In particular, I am going to show that the group of exact auto-equivalences of the derived category of coherent sheaves on a smooth elliptic curve induces an action of the group $\widehat{\mathrm{SL}_2}(\mathbb{Z})$ on the Drinfeld double of the composition subalgebra.
In my third lecture (based on a series of joint articles with Bernd Kreussler), I am going to explain how the classification of indecomposable finite length modules over the ring $\mathbf{k}[[x,y]]/xy$ can be used to describe semi-stable torsion free sheaves on a nodal cubic curve $zy^2 = x^3 + x^2z$ as well as to construct some interesting examples of vector bundles on genus one fibrations.

References

[] (1) K. Brüning, I.Burban Coherent sheaves on an elliptic curve. Interactions between homotopy theory and algebra, 297–315, Contemp. Math., 436, Amer. Math. Soc., Providence, RI, 2007.

[] (2) I.Burban, B.Kreussler, Fourier-Mukai transforms and semi-stable sheaves on nodal Weierstrass cubics, J. Reine Angew. Math., vol. 584, 45-82 (2005).

[] (3) I.Burban, B.Kreussler, Derived categories of irreducible projective curves of arithmetic genus one, Compositio Math., vol. 142, 1231-1262 (2006).

[] (4) I.Burban, O.Schiffmann, On the Hall algebra of an elliptic curve I, math.AG/0505148.

Ringel-Hall Algebras of Cyclic Quivers

Andrew Hubery
NULL
University of leeds
University of leeds
United Kingdom

E-mail address: **ahubery@maths.leeds.ac.uk**

We shall study the Ringel-Hall algebra H_n associated to the category $\mathcal{A}_n$ of finite dimensional nilpotent representations of the cyclic quiver with n vertices. This is naturally a self-dual Hopf algebra and has a Hopf algebra decomposition $H_n \cong C_n \otimes Z_n$, where Z_n is the centre of H_n and C_n is the subalgebra generated by the simple objects of $\mathcal{A}_n$.

There exist canonical Hopf algebra embeddings $\Lambda \to Z_n$ and $C_n \to \mathcal{U}_v(\widehat{\mathfrak{sl}}_n)$, where Λ is Macdonalds ring of symmetric functions and $\mathcal{U}_v(\widehat{\mathfrak{sl}}_n)$ is the quantum affine algebra of type $\widetilde{A}$. Moreover, we can extend this latter to an embedding $\eta_n \colon H_n \to \mathcal{U}_v(\widehat{\mathfrak{gl}}_n)$. In other words, we can realise the Borel subalgebra of $\mathcal{U}_v(\widehat{\mathfrak{gl}}_n)$ as the Ringel-Hall algebra of the category $\mathcal{A}_n$.

Taking these ideas further, we can define embeddings of categories $\mathcal{A}_n \to \mathcal{A}_{n+1}$, inducing algebra embeddings $H_n \to H_{n+1}$. Under η, these correspond to the "upper left corner embeddings $\mathcal{U}_v(\widehat{\mathfrak{gl}}_n) \to \mathcal{U}_v(\widehat{\mathfrak{gl}}_{n+1})$.

Finally, taking colimits, we have the category $\mathcal{A}_\infty := lim \mathcal{A}_n$ and the algebra $H_\infty := lim H_n$, and these constructions commute in the sense that H_∞ is the Ringel-Hall algebra of $\mathcal{A}_\infty$. We can therefore realise the Borel subalgebra of $\mathcal{U}_v(\widehat{\mathfrak{gl}}_\infty)$ as the Ringel-Hall algebra of $\mathcal{A}_\infty$.

References

[] None

Abstracts presented for the general conference in Santos, SP.

RELATIONS BETWEEN TILTING AND STRATIFICATION

Jose Fidel Hernandez Advincula
Departament of Mathemtic
Universidad de La Habana
San Lazaro y L , Vedado
Cuba

E-mail address: **fidel@matcom.uh.cu**

In this work we study the relation between tilting and standard stratification. We recall that for each standardly stratified algebra corresponds a tilting module. We show that the poset given by the different stratifications of one algebra is a subposet of the poset formed by the tilting modules. Also, we show several examples, in particular, in the oriented A_n for $n = 2, 3.4, 5$ all tilting modules are given by stratifications.

References

[AR] ar M. Auslander, I. Reiten, *Aplications of Contravariantly Finite Subcategories*, Advances in Mathematics, 86, 1991, 111-152.

[HU] hu D. Happel and L. Unger, *On a partial order of tilting modules*, Algebr. Represent. Theory, 8(2), 147 -156, 2005.

[HM1] hm1 F. Hernndez Advncula, E. N. Marcos *Algebras which are standardly stratitified in all orders*, RT-MAT 2004-26, Trabalhos do Departamento de Matemtica, IME - USP.

[HM2] hm2 F. Hernndez Advncula, E. N. Marcos *Stratifications of algebras with radical square zero*, RT-MAT 2004-25, Trabalhos do Departamento de Matemtica, IME - USP.

[R] hm3 C. M. Ringel *The category of modules with good filtrations over a quasi-hereditary algebras has almost split sequences*, Mathematische Zeitschrift, Vol. 208, pag. 209-223, 1991.

[X] x Changchang Xi, *Standardly Stratified Algebras and Cellular Algebras*, Mathematical Proceedings of the Cambridge Philosophical Society, 133, pag. 37-53, Cambridge University Press, 2002

Homological characterization of piecewise hereditary algebras

Dieter Happel
Technische Universitaet Chemnitz
Fakultaet fuer Mathematik
Germany

E-mail address: **happel@mathematik.tu-chemnitz.de**

This is a report on joint work with Dan Zacharia

A finite dimensional algebra A is called piecewise hereditary provided its bounded derived category is triangle equivalent to the derived category of a hereditary category. The hereditary categories occurring in this context have been classified. They are module categories of finite dimensional hereditary algebras and the categories of sheaves over a weighted projective line. For an arbitrary finite dimensional algebra B it was suggested by Ringel to consider the strong global dimension. This measures the length of indecomposable complexes whose stalks are projective B-modules. It was conjectured by Kerner, Skowronski, Yamagata and Zacharia that the class of piecewise hereditary algebras coincide with the class of algebras having finite strong global dimension. The aim of the talk will be twofold. First we will recall some homological properties of piecewise hereditary algebras. Then we will outline the proof of the main result. In this we will use a characterization of the derived category of a hereditary category in terms of paths due to Ringel and an interesting lemma on shortening of paths in an arbitrary triangulated category.

References

[] None

Matrix problems and stable homotopy types

Yuriy Drozd
Institute of Mathematics
National Academy of Sciences
Tereschenkivska str. 3
Ukraine

E-mail address: **y.a.drozd@gmail.com**

Let $\mathbf{C}$ be a triangulated category, $\mathbf{A}, \mathbf{B}$ be its fully additive subcategories such that $\mathbf{C}(B, A[1]) = 0$ for all $A \in \mathbf{A}$, $B \in \mathbf{B}$, $\mathbf{A}\dagger\mathbf{B}$ be the full subcategory of $\mathbf{C}$ consisting of all X that occur in the triangles $A \to B \to X \to A[1]$ with $A \in \mathbf{A}$, $B \in \mathbf{B}$. We construct a bimodule category $\mathbf{M}$ and ideals $\mathbf{I} \subset \mathbf{A}\dagger\mathbf{B}$, $\mathbf{J} \subset \mathbf{M}$ such that $(\mathbf{A}\dagger\mathbf{B})/\mathbf{I} \simeq \mathbf{M}/\mathbf{J}$, moreover, $\mathbf{J^2} = 0$.

We use this construction to describe stable homotopy types of polyhedra (finite CW-complexes) $X \in \mathbf{P_n}$, i.e. $(n-1)$-connected of dimension at most $2n-1$, for $n \leq 4$, and for those polyhedra from $\mathbf{P_n}$ that have no torsion in integral homologies, for $n \leq 7$. If $n < 4$ (in torsion free case $n < 7$), the problem is of finite type, for $n = 4$ (in torsion free case $n = 7$) it is tame and the result is formulated in terms of "strings and bands." We also show that for bigger n these problems become *wild*, i.e. contain a classification of representations of an arbitrary finitely generated algebra over a field.

More details can be found in [1,2].

References

[1] Yuriy Drozd, *Matrix problems and stable homotopy types of polyhedra,* Central European J. Math. vol. 2 (2004) 420–447 .

[2] Yuriy Drozd, *On classification of torsion free polyhedra,* Preprint MPI 2005-99, Max-Plank-Institut für Mathematik, 2005.

Skew monoid categories and derived equivalences

Hideto Asashiba
Department of Mathematics, Faculty of Science,
Shizuoka Universtiy
836 Ohya, Suruga-ku
Japan

E-mail address: **shasash@ipc.shizuoka.ac.jp**

In the classification of representation-finite self-injective algebras up to derived equivalences in [2] a covering technique for derived equivalence in [1] developed along the line of the classical covering technique [3] was the main tool. But requirements on categories in the classical covering technique were so strong that they made proofs unnecessarily complicated and applications difficult, in particular, direct application to additive categories was impossible and it was necessary to verify that the group action in consideration was free on isoclasses of indecomposable objects in the bounded homotopy category of finitely generated projective modules. Since this was not always easy to check we had a restriction in application (e.g., we needed an assumption that the group is torsion-free). In the last ICRA I announced an outline of a generalization of the covering technique to remove all classical requirements including the free action condition. This time we deal with applications. In the generalized case orbit categories essentially coincide with skew group categories, and we have the following generalized form of the main theorem of covering technique for derived equivalence with a simplified proof: For algebras (categories) A and B on each of which a group G acts, if there exists a "G-equivariant" derived equivalence between them, then the skew group algebras (categories) $A * G$ and $B * G$ are derived equivalent. In order to apply this we need to know how to compute skew group algebras (categories). We give a general way to compute skew monoid algebras (categories) using quivers with relations (cf. [4]), which enables us to compute examples of derived equivalent skew group algebras.

References

[1] Asashiba, H.: *A covering technique for derived equivalence*, J. Alg., bf 191 (1997), 382–415.

[2] Asashiba, H.: *The derived equivalence classification of representation-finite selfinjective algebras*, J. Alg., **214** (1999), 182–221.

[3] Gabriel, P.: *The universal cover of a representation-finite algebra*, Lecture Notes in Mathematics, Vol. **903**, Springer-Verlag, Berlin/New York (1981), 68–105.

[4] Reiten, I. and Riedtmann, Ch.: *Skew group algebras in representation theory of artin algebras*, J. Alg **92**, (1985) 224–282.

Representations of Finite Posets

Volodymyr Kyrychenko
Mechanics and Mathematics
Kyiv Taras Shevchenko National University
Volodymyrska, 64
Ukraine

E-mail address: **vkir@univ.kiev.ua**

(jointly with V.Bondarenko, N.Gubareni and M.Khybyna)

Let $\mathcal{P} = \{p_1, p_2, \dots, p_n\}$ be a poset with ordering relation $\preceq$. Denote by $\mathcal{A}(\mathcal{P})$ the set of all antichains of $\mathcal{P}$ of length $l \geq 0$. We assume that the antichain of length 0 is an empty set, and we denote this antichain as 0. We identify antichains of length 1 with the elements themselves. We define the order relation $\leq$ in $\mathcal{A}(\mathcal{P})$ as follows. If $X, Y \in \mathcal{A}(\mathcal{P})$, then $X \leq Y$ if and only if for any $a \in X$ there exists $b \in Y$ such that $b \preceq a$. We also assume that $0 < X$ for any $X \neq 0$.

Denote $A(\mathcal{P}) = \mathcal{A}^1(\mathcal{P})$ and $\mathcal{A}^m(\mathcal{P}) = \mathcal{A}(\mathcal{A}^{m-1}(\mathcal{P}))$

We say that a poset $\mathcal{P}$ is a garland if for any $p \in \mathcal{P}$ there exists at most one $q \in \mathcal{P}$ such that p and q are incomparable.

Proposition. *Let $\mathcal{P}$ be a poset of width 2 and let $\mathcal{A}^m(\mathcal{P})$ be a poset of width 2 for all $m > 1$. Then $\mathcal{P}$ is a garland. Conversely, if $\mathcal{P}$ is garland then $\mathcal{A}^m(\mathcal{P})$ be a poset of the width 2 for all $m > 1$.*

Let C_m be a chain of m elements. Consider the cardinal square $C_m \sqcup C_m$, where C_m is a chain of m elements. Note that the width of $\mathcal{A}(C_m \sqcup C_m)$ equals $m + 1$. We use the construction $\mathcal{P} \to \mathcal{A}(\mathcal{P})$ for the representation theory of finite posets.

References

[] None

Representation finite cluster-tilted algebras and bound modulated quivers

Nguefack Bertrand
Départ. de Mathématiques, Fac. des sciences
Université de Sherbrooke
2500 boul. Université
Canada

E-mail address: **ng.bertd@yahoo.ca**

We investigate representation finite cluster-tilted algebras over any field using bound modulated quivers. For a Dynkin valued graph $\Delta \in \{\mathbb{A}_n, \mathbb{B}_n, \mathbb{C}_n\}$, we define a special valued graph $\mathbb{T}_\Delta$ called the *triangle-decorated tree of class* Δ. We then start by proving a structure theorem which explicitly describes all valued quivers in the mutation class of Δ in terms of full valued subgraphs of $\mathbb{T}_\Delta$. We next obtain a "characterization theorem for representation finite cluster-tilted algebras" which generalizes a similar result for simply-laced cluster-tilted algebras. We also give necessary and sufficient criterion for a bound modulated quiver algebra to be cluster-tilted of type $\mathbb{A}_n$, $\mathbb{B}_n$, $\mathbb{C}_n$, $\mathbb{F}_4$ or $\mathbb{G}_2$. In particular, any representation finite cluster-tilted algebra $\widetilde{\mathcal{A}}$ is uniquely determined by its modulated quiver. Also, the representation type of $\widetilde{\mathcal{A}}$ depends only on its valued quiver.

References

[] none

Chevalley Groups and Hyperalgebras

Tiago Macedo
IMECC
UNICAMP
Brazil

E-mail address: **ra025318@ime.unicamp.br**

We will present results relating the Distribution algebra of Chevalley Groups with the so called hyperalgebras. These hyperalgebras are Hopf algebras built from Kostant integral form of semisimple Lie algebras by reduction mod p.

References

[1] J. Humphreys, Linear algebraic groups, GMT21 Springer (1981).

[2] J. Jantzen, Representation of algebraic groups, Mathematical Surveys and Monographs **107**, AMS (2003).

A right normed basis for free Lie algebras

Evgeny Chibrikov
IME
USP
Rua Corinto 431 22 BL A
Brazil

E-mail address: **chibr@gorodok.net**

In this paper we construct a basis of a free Lie algebra that consists of right normed words, i.e. the words that have the following form: $[a_{i_1}[a_{i_2}[\ldots[a_{i_{t-1}}a_{i_t}]\ldots]]]$, where a_{i_j} are free generators of the Lie algebra.

References

[] None

Almost hereditary noetherian rings

Otto Kerner
Mathematisches Insititut
H.-Heine-Universitaet
Universitaetsstr. 1
Germany

E-mail address: **kerner@math.uni-duesseldorf.de**

When D. Happel, I. Reiten and S. Smalødeveloped the concept of quasi-tilted algebras, they showed that for an R-artin-algebra A the following conditions are equivalent: (i) A is almost hereditary. (ii) There exists a split torsion pair $(\mathcal{X}, \mathcal{Y})$ in mod$A$, such that $\mathcal{Y}$ consists of modules of proj. dimension ≤ 1 and contains A. (iii) A is isomorphic to the endomorphism ring of a tilting object T in a locally finite hereditary abelian R-category.

In collaboration with J. Stovicek and J. Trlifaj we generalised this result to right noetherian rings.

References

[] None

NULL

Olivier Mathieu
NULL
Universite de Lyon
UFR de mathematiques
France

E-mail address: **mathieu@math.univ-lyon1.fr**

On varieties determined by their jets (joint with R. Gurjar).

Let X be an affine variety over the complex field $\mathbf{C}$, and let Y be a subvariety defined by the ideal I of $\mathbf{C}[X]$ and let $\hat{X}$ be the infinitesimal neighborhood of Y in X. By definition, $\mathbf{C}[\hat{X}]$ is the I-adic completion of $\mathbf{C}[X]$.

Usually, one cannot recover X from $\hat{X}$. For example, if X is smooth of dimension n and Y is a point, then $\mathbf{C}[\hat{X}] \simeq \mathbf{C}[[x_1, \ldots, x_n]$. However, in the case where a reductive group G acts over X and when Y is the unique closed orbit under the action, then $\hat{X}$ determined X. More precisely:

Theorem *Let G (respectively G) be a reductive groups acting on X (respectively$X$). with a unique closed orbit Y (respectively Y). If $\hat{X}$ and $\hat{X}$ are isomorphic, then X and X are isomorphic.*

Some consequences of this result are given.

References

[] None

Actions on inverse semigroups on algebras

Wagner de Oliveira Cortes
NULL
UFRGS
Av Bento Goncalves, 9500
Brazil

E-mail address: **wocortes@gmail.com**

We study that if S is a faithfully projective R-algebra and H is a finite inverse semigroup acting on S as R-linear maps such that the fixed subring $S^H = R$, then any partial isomorphism between ideals of S which are generated by central idempotents can be obtained as restriction of an R-automorphism of S and there exists a finite subgroup of automorphisms G of S with $S^G = R$.

This article was made with Miguel Ferrero, Antonio Paques e Dirceu Bagio.

References

[] None

SAGBI bases for the kernel of a locally nilpotent k-derivation in $k[x_1, .., x_n]$.

Angelo Calil Bianchi
UNICAMP
Campinas, Brazil

E-mail address: **angelo@ime.unicamp.br**

The objective of this work is to present the algebraic theory of locally nilpotent k-derivations in $k[x_1, .., x_n]$, based on a Zariski's automorphism of this ring, and an algorithmic way to determine the generators and a SAGBI base of its kernels.

References

[] Freudenburg, G., *Algebraic Theory of Locally Nilpotent Derivations*, Springer, 2006.

Triangular Matrix and Koszul Algebras

Gustavo Montaño-Bermúdez
Facultad de Ciencias
Universidad Autónoma del Estado de México
Av. Instituto Literario No.100 Col. Centro
Toluca, México, México

E-mail address: **gmb@uaemex.mx**

(Jointly with R. Martínez-Villa)

It is well known that Koszul algebras are quadratic and monomial quadratic algebras [Green-Zacharia] and quadratic algebras of global dimension two are Koszul. It was also proved [Green-Huang] that algebras with a quadratic Groebner basis are Koszul, however there is no general characterization of Koszul algebras, hence it is of interest to construct new Koszul algebras from given ones.

We consider triangular matrix algebras

$$\Lambda = \begin{bmatrix} R & 0 \\ M & T \end{bmatrix},$$

with R and T $\mathbb{K}$-algebras and M a T-R-bimodule.

The aim of this talk is to give sufficient conditions on R, T and M in order to obtain a Koszul algebra Λ.

References

[] None

Connected gradings and fundamental group

Claude CIBILS

Mathematics
Universit Montpellier 2
Pl. E. Bataillon
France

E-mail address: **Claude.Cibils@math.univ-montp2.fr**

Joint work with Maria Julia Redondo and Andrea Solotar

The fundamental group of a category over a field is the inverse limit of the groups providing connected gradings of the category. This invariant is related to Galois coverings of the category and is not a Morita invariant. We consider the thin categories of Galois coverings and of connected gradings in order to show that they are isomorphic, providing this way an intrinsic formulation of results obtained by E.L. Green and E.N. Marcos for a category presented as a path category of a quiver with relations. Then we will prove that algebras of matrices do not have an universal grading (or equivalently an universal cover), nevertheless for an algebraically closed field of good characteristic there exist precisely two source objects in the category of gradings. This allows to compute explicitly the fundamental group of matrices of sizes 2 and 3, making use of classification results by C. Boboc, S. Dăscălescu and R. Khazal.

References

[]

Mutation of Cluster-tilting Objects and Potentials

David Smith

Mathematics
Bishops University
2600 College Street
Canada

E-mail address: **dsmith@ubishops.ca**

Motivated by the theory of cluster algebras initiated by Fomin and Zelevinsky [3], Buan, Marsh and Reiten [1] introduced via the notion of cluster categories the so-called cluster-tilted algebras. Through further papers, the ordinary quivers of cluster-tilted algebras were shown to be obtained by sequences of Fomin-Zelevinsky quiver mutations on acyclic quivers; but the nature of the relations on these quivers remained unknown.

On the other hand, there is a recent theory of mutation of quivers with potential initiated by Derksen, Weyman and Zelevinsky [2], and to which are associated algebras called Jacobian algebras. In this talk, we discuss the strong relationship between mutation of cluster-tilting objects in triangulated 2-Calabi-Yau categories and mutation of quivers with potentials. In particular, we show that cluster-tilted algebras are Jacobian algebras, solving the problem of finding the relations on cluster-tilted algebras.

This talk is based on a recent joint work with A. Buan, O. Iyama and I. Reiten.

References

[1] Aslak Bakke Buan, Robert Marsh and Idun Reiten, *Cluster-tilted algebras*, Trans. Amer. Math. Soc. 359 (2007), no. 1, 323–332 (electronic).

[2] Harm Derksen, Jerzy Weyman and Andrei Zelevinsky, *Quivers with potentials and their representations I: Mutations*, to appear in Selecta Math.

[3] Sergey Fomin and Andrei Zelevinsky, *Cluster algebras. I. Foundations.*, J. Amer. Math. Soc. 15 (2002), no. 2, 497–529 (electronic).

Olena Drozd-Korolova
Department of Mechanics and Mathematics
National Taras Shevchenko University of Kyiv
64, Volodymyrska street, Kyiv
Ukraine

E-mail address: **olena.drozd@gmail.com**

Decorated generalized bunches of chains

We consider a class of matrix problems, which contains both generalized bunches of chains (or Gelfand problems, or clans) [3] and decorated bunches of chains that appear in the theory of Cohen–Macaulay modules over non-isolated singularities [2].

Definition Let $\{\mathcal{E}, \mathcal{F}, <, \sim, -, E_c, F_c\}$ be a generalized bunch of chains (GBC) over a field K in the sense of [3]. A *decoration* of this GBC consists of:

1. A discrete valuation on K and its prolongations to every skewfiled E_c, F_c; we denote by K^*, E_c^*, F_c^* the corresponding discrete valuation rings.

2. A subset D of pairs (x, y) such that $x \leq y$; we call these pairs *decorated*. Especially, we call x *decorated* if so is the pair (x, x).

The following condition must hold:

If $x \leq y \leq z$ and the pair (x, z) is decorated, then both pairs (x, y) and (y, z) are also decorated. Especially, if x occur in a decorated pair it is decorated itself.

We define *representations of a decorated GBC* by analogy with [3] and [2]. The main distinction is that for decorated pairs (x, y) the corresponding elementary transformations are only possible with coefficients from E_c^* or F_c^*. Then, following [1], we elaborate an algorithm of reduction and give a combinatorial description of indecomposable representations in terms of *strings* and *bands*.

I will also present some examples of the applications.

References

[1] V. M. Bondarenko, *Representations of bundles of semichained sets and their applications*, St. Petersburg Math. J. 3 (1992), 973–996.

[2] I. Burban and Y. Drozd. *Cohen–Macaulay modules over non-isolated surface singularities*, Preprint, 2007.

[3] O. Drozd. *Representations of generalized bunches of chains*, Algebraic Structures and Their Applications. Kyiv, 2002, . 224–237.

On the relative socle for stratifying systems

Corina Saenz

Matematicas

Facultad de Ciencias

Circuito Exterior, Ciudad Universitaria

Mexico

E-mail address: **ecsv@lya.fciencias.unam.mx**

Joint with: M. Lanzilotta, E. Marcos, O. Mendoza

Let A be an algebra and M be a finitely generated A-module. In the class of all the sub-objects of M, we have a pre-order relation $\leq$. That is, given two monomorphisms $f : X \to M$ and $g : Y \to M$, it is said that $f \leq g$ if there is a morphism $h : X \to Y$ such that $gh = f$. This pre-order induces an equivalence relation in the class of all sub-objects of M. Indeed, $f \sim g$ if and only if $f \leq g$ and $g \leq f$. So, the pre-order becomes a partial order on the set of equivalence classes of sub-objects of M.

Consider a stratifying system $(\Theta, \leq)$ of size t, $M \in \mathcal{F}(\Theta)$, the category of finitely generated left modules having a Θ-filtration, and $f : N \to M$ a Θ-monomorphism. If $g : X \to M$ is a sub-object of M which is equivalent to f, then g is also a Θ-monomorphism. Therefore, the notion of Θ-monomorphism is compatible with the pre-order relation considered above. On the other hand, we have the notion of Θ-length of $M \in \mathcal{F}(\Theta)$, which is given by $\ell_\Theta(M) := \sum_{i=1}^{t} [M : \Theta(i)]$, where $[M : \Theta(i)]$ is the multiplicity of $\Theta(i)$ in M.

Given a module M in the category $\mathcal{F}(\Theta)$, we study the concept of the relative socle of M. We do so, under two different approaches, which are:

(Ap.1) as a Θ-semisimple subobject of M having maximal Θ-length,

(Ap.2) as a maximal Θ-semisimple subobject of M with respect to the order $\leq$ introduced above.

References

[] M.Lanzilotta, E.Marcos, O.Mendoza, C.Sáenz, On the relative socle for stratifying systems. Work in progress.

Hochschild homology and global dimension

Petter Andreas Bergh

NTNU

Norway

E-mail address: **bergh@math.ntnu.no**

In his 1989 paper on Hochschild cohomology, Happel raised the following question: if the Hochschild cohomology of a finite dimensional algebra vanishes in high degrees, then does the algebra have finite global dimension? This was answered negatively in a paper by Buchweitz, Green, Madsen and Solberg, where a counterexample was given. However, the Hochschild homology version of the question, a conjecture given by Han, is still open. We give a positive answer to this conjecture for local graded algebras, Koszul algebras and cellular algebras. The proof uses Igusas formula for relating the Euler characteristic of the relative cyclic homology to the graded Cartan determinant.

This is joint work with Dag Madsen.

References

[1] P. A. Bergh, D. Madsen, *Hochschild homology and global dimension*, arXiv:0803.3550v1.

[2] R.-O. Buchweitz, E. Green, D. Madsen, Ø. Solberg, *Finite Hochschild cohomology without finite global dimension*, Math. Res. Lett. 12 (2005), no. 5-6, 805-816.

[3] Y. Han, *Hochschild (co)homology dimension*, J. London Math. Soc. (2) 73 (2006), no. 3, 657-668.

[4] D. Happel, *Hochschild cohomology of finite-dimensional algebras*, in Lecture Notes in Mathematics 1404, Springer-Verlag (1989), 108-126.

3-equipped posets and their representations and corepresentations

Ivon Dorado

Departamento de Matemáticas

Universidad Nacional de Colombia

Bogotá
Colombia

E-mail address: **iadoradoc@bt.unal.edu.co**

The research concerns some problems of the representation theory of posets with additional structures. Our main objective are posets equipped with more than one order relation.

Recall that, in a series of papers by Zavadskij and his students, there were introduced and studied *equipped posets*, with order relations of two kinds: weak and strong. Their representations (and corepresentations) determine some matrix problems of mixed type over quadratic field extensions.

Extending this ideas, we define now *3-equipped posets* (with order relations of three kinds) and their representations and corepresentations over a purely inseparable cubic field extension $F \subset G$, in characteristic 3.

We mainly study the infinite-representation case dealing with the matrix problems of mixed type over the pair (F, G) which correspond to classification of indecomposables. For some critical 3-equipped posets, their representations and corepresentations are completely classified and described in evident matrix form. Meanwhile for the others, the task is reduced to some matrix problems that contain the pseudolinear pencil problem as a subproblem.

References

[] None

Weight modules over affine Lie algebras

Ivan Dimitrov
Queen's University
Canada

E-mail address: **dimitrov@mast.queensu.ca**

We classify all irreducible weight modules with finite dimensional weight spaces over affine Lie algebras. This problem has been studied extensively in the last twenty years. Futorny introduced the notion of a dense weight module and classified all non-dense irreducible modules. Chari and Pressley showed that every integrable irreducible weight module with finite dimensional weight spaces is either a loop module or a highest weight module. They also studied more general loop modules. The above mentioned works of Futorny, Chary, and Pressley produced a list of irreducible weight modules with finite dimensional weight spaces which Futorny conjectured to be complete. In this talk I will talk about the proof of this conjecture.

This is a joint work with Dimitar Grantcharov.

Twisted Representations of Quivers

Daniela Prata

IMECC
UNICAMP
Brazil

E-mail address: **daniela.mprata@gmail.com**

In this work we introduce the concept of twisted representations of a quiver and we prove an equivalence between the category of twisted representations of a quiver Q, and the category of representations of a quiver $\tilde{Q}$, where $\tilde{Q}$ depends on Q and on the twisting factors.

References

[] P. B. Gothen , A. D. King, *Homological algebra of twisted quiver bundles*, J. London. Math. Soc. **71**, (2005) 85-99.

Gentle algebras and derived equivalence

Diana Avella Alaminos
Matemáticas
Facultad de Ciencias UNAM
Av. Universidad 3000 Copilco Coyoacán México DF
México

E-mail address: **avella@matem.unam.mx**

Joint work with Christof Geiss.

We study the classification of gentle algebras according to derived equivalence, which is well understood in the case of gentle algebras whose associated quiver has just one cycle.

We define derived equivalent invariants for gentle algebras, constructed in an easy combinatorial way from the quiver with relations defining these algebras. They consist of pairs of natural numbers and contain important information about the algebra and the structure of the stable Auslander-Reiten quiver of its repetitive algebra.

References

[1] D. Avella-Alaminos, C. Geiss, *Combinatorial derived invariants for gentle algebras*, J. Pure Appl. Algebra **212** (2008), no. 1, 228-243.

Rank functors and representation rings of quivers

Ryan Kinser
Department of Mathematics
University of Michigan
530 Church St
United States

E-mail address: **kinser@umich.edu**

Abstract: I will discuss the recently constructed functor which takes a representation of an arbitrary (finite) quiver Q, and returns a representation of Q for which the maps over all arrows are isomorphisms. The common dimension of the resulting vector spaces at each vertex is a numerical invariant of the representation. Combining these functors with pulling back representations along well chosen maps of directed graphs allows one to construct other numerical invariants of representations. These include, as the simplest cases, the dimension vector of a representation, the ranks of any composition of maps, dimensions of intersections of images, and so forth. We call the functors giving rise to these invariants "rank functors", although in general they measure something more complicated than the rank of any one map.

There is a natural tensor product on representations of Q, which allows one to construct a representation ring R(Q) a la Grothendieck. The rank functors above commute with direct sum and tensor product of representations (addition and multiplication in R(Q)), and fix the identity, hence induce ring homomorphisms from R(Q) to the integers, called rank functions. In more recent work, when Q is a tree quiver with a unique sink, I use combinatorial methods to construct all rank functions on Q and show that ring R(Q), modulo its ideal of nilpotents, is finitely generated as an abelian group.

References

[1] Ryan Kinser, *The rank of a quiver representation*, arXiv:0711.1135

On generating of homotopy categories of complexes

Jan Šťovíček
NTNU
Norway

E-mail address: **stovicek@math.ntnu.no**

When dealing with triangulated categories which are not small, it is a very useful property if they are compactly generated (such as the unbounded derived category of a module category). It was recently studied by Holm, Jørgensen, Krause and Neeman whether or when different homotopy categories of complexes, such as $K(\text{Mod-}R)$, $K(\text{Flat-}R)$, $K(\text{Proj-}R)$ or $K(\text{Inj-}R)$, are compactly generated.

In this talk, I will outline a general method for studying this problem. I will employ Neeman's concept of a well-generated triangulated category which generalizes compactly generated categories. I will show that the homotopy category $K(\text{Mod-}R)$ is "locally" well-generated. However, if $\mathcal{A}$ is an additive category with coproducts, then $K(\mathcal{A})$ is truly well-generated only if a rather restrictive condition is satisfied.

As a result, a complete answer to the question about compact generatedness of $K(\text{Mod-}R)$ for any ring R is given, and the cases of $K(\text{Flat-}R)$ and $K(\text{Proj-}R)$, where the situation is more complicated, are explained. This way, many natural examples of non-well-generated categories are provided and a general method for detecting more such examples among algebraic triangulated categories is shown.

References

[1] H. Holm and P. Jørgensen, *Compactly generated homotopy categories*, Homology, Homotopy Appl. **9** (2007), 257–274.

[2] H. Krause, *The stable derived category of a noetherian scheme*, Compos. Math. **141** (2005), 1128-1162.

[3] A. Neeman, *The homotopy category of flat modules, and Grothendieck duality*, preprint.

Links of faithful partial tilting modules

Luise Unger
Faculty of Mathematics and Computer Science
Fernuniversity Hagen
Lützowstr. 125, D-58084 Hagen
Germany

E-mail address: **luise.unger@fernuni-hagen.de**

This is a report on some recent joint work with Dieter Happel.

Let Λ be a basic, hereditary, finite dimensional algebra over an algebraically closed field k. We denote by $\overrightarrow{\mathcal{K}}_\Lambda$ the quiver of tilting modules over Λ. For a direct summand M of a multiplicity free Λ-tilting module let $\overrightarrow{\mathrm{lk}}(M)$ be the full subquiver of $\overrightarrow{\mathcal{K}}_\Lambda$ with vertices T such that M is a direct summand of T. We call $\overrightarrow{\mathrm{lk}}(M)$ the link of M. In general $\overrightarrow{\mathrm{lk}}(M)$ is not connected.

We indicate the proof that $\overrightarrow{\mathrm{lk}}(M)$ is connected provided M is faithful.

References

[] None

On the representation dimension of one-point extensions

Manuel Saorín
NULL
Universidad de Murcia
Campus de Espinardo
Spain

E-mail address: **msaorinc@um.es**

In this talk, we will present some results obtained in ongoing work with Oppermann, Reiten and Solberg. The main problem addressed in the talk will be that of finding sufficient conditions for a one-point extension $H[M]$ to have representation dimension less or equal than 3, where H is a hereditary finite-dimensional algebra.

References

[] None

On corepresentations of one-parameter equipped posets over a quadratic field extension

Claudio Rodriguez
Departamento de Matemáticas
Universidad Nacional
Bogotá
Colombia

E-mail address: **crodriguezbe@unal.edu.co**

We consider equipped posets (i.e. posets with two kinds of order relations) and investigate their corepresentations over a quadratic field extension $\mathbb{F} \subset \mathbb{G}$.

Representations of one-parameter equipped posets over the classical field extension $\mathbb{R} \subset \mathbb{C}$ were described completely in matrix form in [1]. Their classification led to some matrix problems of mixed type over the pair $(\mathbb{R}, \mathbb{C})$. On the other hand, recently in [2] there were introduced and studied also corepresentations of equipped posets. Their classification corresponds to similar matrix problems of mixed type which are in some intuitive sense dual to the mentioned ones for representations. Nevertheless, there is no yet any formal construction reducing one type of problems to another one.

Having in mind the objective to classify in the future all corepresentations of one-parameter equipped posets over an arbitrary quadratic extension $\mathbb{F} \subset \mathbb{G}$, we establish some necessary properties of corepresentations based on characteristics of the critical subposets. We also classify completely in matrix form all the indecomposable corepresentations of several critical and faithful one-parameter equipped posets. One of the essential investigation tools is the algorithm of differentiation $\widehat{VII}$ constructed in [2].

References

[1] A.V. Zabarilo, A.G. Zavadskij, One-parameter equipped posets and their representations,Functional Anal. Appl., vol. 34(2000), no.2, 138–140.

[2] C. Rodrguez, A.G. Zavadskij, On corepresentations of equipped posets and their differentiation,Revista Colombiana de Matemticas, vol. 41(2007)117–142.

On sums of figurate numbers by using techniques of poset representation theory

Agustin Moreno Cañadas
Matematicas
UNIVERSIDAD NACIONAL DE COLOMBIA
Bogota
Colombia

E-mail address: **amorenoca@unal.edu.co**

This talk is dedicated to the applications of the poset representation theory to the number theory. We recall that the representation theory of posets was developed in the 70's by Nazarova and Roiter. The main aim of this study was to determine indecomposable matrix representations of a given poset over a fixed field. Soon after the discovery of matrix representations of a poset, Gabriel introduced the concept of a filtered linear representation of a poset in connection with the investigation of oriented graphs having finitely many isomorphism classes of indecomposable linear representations [3].

In this talk we shall describe how we can use differentiation algorithms (which have been successfully applied in determining the representation type (finite or infinite) of posets and in the classification of indecomposable poset representations), as well as representations over the set of natural numbers of a poset, in order to obtain solutions of some open problems concerning figurate numbers [1,2]. In particular, we present criteria for natural numbers which are the sum of three octahedral numbers (in connection with the Pollock's conjecture on tetrahedral and octahedral numbers), three polygonal numbers of positive rank (note for example that the problem of representing a natural number as sums of three nonvanishing squares is still an unsolved problem [1]) or four cubes with two of them equal. Some identities of the Rogers- Ramanujan type involving this class of numbers are also obtained with the help of poset representation theory.

References

[1] E. Grosswald, representations of integers as sums of squares, Springer-Verlag, New York, 1985.

[2] R. Guy, Unsolved Problems in Number Theory, 3rd ed., Springer-Verlag, New York, 2004.

[3] D. Simson, Linear Representations of Partially Ordered Sets and Vector Space Categories, Gordon and Breach, London, 1992.

Some classification problems of linear algebra and representations of equipped posets

Alexander Zavadskij
Departamento de Matemáticas
Universidad Nacional de Colombia
Bogotá
Colombia

E-mail address: **ozavadskyy@unal.edu.co**

An equipped poset is a poset with some additional binary relation. Representations of equipped posets over a quadratic field extension $F \subset G$ lead to certain natural matrix problems of mixed type over the pair (F, G).

Our objective is to deal with the representation infinite (tame) equipped posets. In case of the classical quadratic field extension $\mathbb{R} \subset \mathbb{C}$, we already have (as a result of the previous investigations) the developed differentiation technique and the representation type criteria for equipped posets to be one-parameter, tame, of finite growth.

To extend the theory to an arbitrary quadratic extension $F \subset G$, one needs in particular to classify the indecomposable representations of the natural G-G-bimodule $W = G \otimes_F G$. We do it [2] for a separable (inseparable) extension $F \subset G$ reducing the task to the semilinear (pseudolinear) pencil problem, solved by Djoković in 1978 (Sergeichuk in 1989). We also obtain a simplified canonical form of indecomposables of the bimodule W in characteristic $\neq 2$. The construction involves a special technique of transformations of polynomials on the base of some integer matrix sequence [1]. The results find applications in constructing generalized differentiation algorithms and classifying indecomposables.

References

[1] A.G. Zavadskij, On the Kronecker problem and related problems of linear algebra, Linear Algebra Appl., **425** (2007), 26–62.

[2] A.G. Zavadskij, A matrix problem over a central quadratic skew field extension, Linear Algebra Appl., **428** (2008), 393–399.

Semi-Invariants of Tubular Algebras

Kristin Webster
NULL
Northeastern University
360 Huntington Ave
United States

E-mail address: **webster.kr@neu.edu**

Let KQ/I be the path algebra of the quiver $Q = (Q_0, Q_1)$ with the ideal I of admissible relations. There is a theorem of Derksen and Weman stating that the semi-invariants for quivers with relations are also generated by the determinantal semi-invariants of Schofield. Using this theorem, we can describe the rings of semi-invariants of tubular algebras. In particular, we are interested in the semi-invariants $SI(Q/I, \beta)$ where β is a dimension vector of a regular KQ/I module. This talk will also include a brief explanation of how shrinking functors are used to show isomorphism of rings of semi-invariants for 1-parameter families of modules.

References

[] [DW1]H. Derksen, J. Weyman, Semi-invariants of quivers and saturation for Littlewood-Richardson coefficients, J. Amer. Math. Soc. 13 (2000), no. 3, 467–479,

[] [DW2]H. Derksen, J. Weyman, On the canonical decomposition of quiver representations, Compositio Math., 133, 245-265 (2002).

[] [DW3]H. Derksen, J. Weyman, Rings of semi-invariants for quivers with relations, J. Algebra, 258, 216-227 (2002).

[] [R1] C.M. Ringel, The rational invariants of tame quivers, Invent. Math. 58(1980),217-239,

[] [R2] C.M. Ringel, Tame algebras and integral quadratic forms, Lecture Notes in Math. 1099, Springer 1984,

[] [S1] A. Schofield, Semi-invariants of quivers, J. London Math. Soc. 43 (1991), 383-395,

[] [SV] A. Schofield, M. Van den Bergh, *Semi-invariants of quivers for arbitrary dimension vectors*, Indag. Math. (N.S.) **12** (2001), no.1, 125–138.

[] [SW1] A. Skowro ski, J. Weyman, The algebras of semi-invariants of quivers, preprint, 1999,

[] [SW2]A. Skowro ski, J. Weyman, The algebras of semi-invariants of canonical algebras, preprint, 1999,

Finite-dimensional representations of hyper loop algebras

Dijana Jakelic
Department of Mathematics, Statistics, and Computer Science
University of Illinois at Chicago
851 S. Morgan Street, 322 SEO
United States

E-mail address: **dijana@math.uic.edu**

Hyper loop algebras are certain Hopf algebras associated to affine KacMoody algebras. We will focus on finite dimensional representations of hyper loop algebras over arbitrary fields. The main results concern the classification of the irreducible representations, their tensor products, the construction of the Weyl modules, and base change. Several of the results are related to the study of irreducible representations of polynomial algebras and Galois theory. This is a joint work with Adriano Moura.

References

[] None

Non-commutative curves and tilting

Igor Burban and Yuriy Drozd

E-mail address: **burban@math.uni-bonn.de, drozd@imath.kiev.ua**

A *non-commutative curve* is a pair $(X, \mathcal{A}_X)$, where X is an algebraic curve over a field $\Bbbk$ and $\mathcal{A}_X$ is a sheaf of $\mathcal{O}_X$-algebras, coherent as a sheaf of $\mathcal{O}_X$-modules. We always suppose that this curve is *reduced*, i.e. $\mathcal{A}_X$ has no nilpotent ideals. An *Auslander curve* of such a (non-commutative, reduced) curve is a curve $(X, \mathcal{E}nd_{\mathcal{A}_X}(\mathcal{M}))$, where $\mathcal{M}$ is a coherent torsion free sheaf of $\mathcal{A}_X$-modules such that, for each point $x \in X$, the stalk $\mathcal{M}_x$ is an additive generator of the category of torsion free (finitely generated) $\mathcal{A}_{X,x}$-modules. Then $\mathrm{gl.dim}\, \mathcal{A}_X \leq 2$ (exactly 2 if the curve is not smooth, i.e not all stalks $\mathcal{A}_{X,x}$ are hereditary).

Suppose that X is a rational (commutative) curve, which only has simple nodes and cusps as singularities, $\pi : \tilde{X} \to X$ is its normalization, $\mathcal{O} = \mathcal{O}_X$ and $\tilde{\mathcal{O}} = \pi_*(\mathcal{O}_{\tilde{X}})$. Set $\mathcal{M} = \mathcal{O} \oplus \tilde{\mathcal{O}}$ and $\mathcal{A} = \mathcal{E}nd_{\mathcal{O}}(\mathcal{M})$.

Then $(X, \mathcal{A}_X)$ is an Auslander curve of $(X, \mathcal{O}_X)$. We construct a tilting complex $\mathcal{T}$ in the derived category $\mathcal{D}^b(\mathrm{Coh}(\mathcal{A}_X))$ and calculate its endomorphism algebra Λ_X. Namely, let $\{X_1, X_2, \dots, X_s\}$ be the irreducible components of $\tilde{X}$ (all of them are identified with the projective line), $\{x_1, x_2, \dots, x_m\}$ be the nodes of X and $\{y_1, y_2, \dots, y_n\}$ be its cusps. Let also $\pi^{-1}(x_j) = \{x'_j, x''_j\}$, where $x'_j \in X_{i'(j)}$ and $x''_j \in X_{i''(j)}$ (possibly $i''(j) = i'(j)$), while $\pi^{-1}(y_k) \in X_{i(k)}$. Then $\Lambda_X = \Bbbk Q/I$, where Q is the quiver with the set of vertices

$$\{v_i, v'_i, x_j, y_k \,|\, 1 \leq i \leq s,\ 1 \leq j \leq m,\ 1 \leq k \leq n\}$$

and the set of arrows

$$\{a'_j : x_j \to v_{i'(j)},\ a''_j : x_j \to v_{i''(j)}, b'_k, b''_k : y_k \to v_{i(k)}, c_i, d_i : v_i \to v'_i\}.$$

The ideal I is generated by the relations

$$\begin{array}{l}
(\eta'_j c_{i'(j)} - \xi'_j d_{i'(j)})a'_j \text{ if } x'_j = (\xi'_j : \eta'_j),\\
(\eta''_j c_{i''(j)} - \xi''_j d_{i''(j)})a''_j \text{ if } x''_j = (\xi''_j : \eta''_j),\\
(\eta_k c_{i(k)} - \xi_k d_{i(k)})b'_k \text{ and } (\eta_k c_{i(k)} - \xi_k d_{i(k)})b''_k - b'_k \text{ if } \pi^{-1}(y_k) = (\xi_k : \eta_k).
\end{array}$$

In particular, $\mathrm{gl.dim}\,\Lambda_X = 2$ and $\mathcal{D}^b(\mathrm{Coh}(\mathcal{A}_X)) \simeq \mathcal{D}^b(\Lambda_X\text{-mod})$, while the category $\mathcal{D}^-(\mathrm{Coh}(X))$ embeds into $\mathcal{D}^-(\Lambda_X\text{-mod})$ as a full triangulated subcategory.

Note that the algebra Λ_X is *gentle* if X is either a *chain* or a *cycle* of projective lines. Since gentle algebras are known to be derived-tame, this gives a new proof of tameness of the category of coherent sheaves $\mathrm{Coh}(X)$ and its perfect derived category. Other way arround, such a geometric description of the obtained gentle algebras leads to interesting corollaries about their Serre functor and the full subcategory of *band complexes*.

This result can be generalized to non-commutative curves such that all their singularities are *nodal algebras*.

Partial orders in the representations of algebras.

Sverre O. Smalø
NULL
NTNU
Trondheim
Norway

E-mail address: **sverresm@math.ntnu.no**

For a finitely generated algebra A over an algebraically closed field k, and a natural number d, the space of d-dimensional representations, $rep_d A$, is the space of k-algebra homomorphisms from A to $M_n(k)$, the algebra of $d \times d$-matrices over k. On this space one can consider for each natural number m the preorder $\leq_m$ given by $f_M \leq_m f_N$ if the dimension of the kernel of the matrix $(f_M(a_{i,j}))$ is less than or equal to the dimension of the kernel of the matrix $(f_N(a_{i,j}))$ for all $m \times m$-matrices $(a_{i,j})$ with entries from A.

In this lecture it will be shown that this preorder is a partial order for a given d when m is big enough. Also relations to other partial orders on the space of representations will be given.

References

[] None

Classification of the tame dimension vectors for wild trees and construction of the one parameter families of indecomposable representations

Angela Holtmann
Fakultät für Mathematik
Universität Bielefeld
Universitätsstr. 25
Germany

E-mail address: **aholtman@math.uni-bielefeld.de**

It is well known that the indecomposable representations of wild quivers depend on arbitrary many parameters making the classification of all indecomposable representations for wild quivers impossible. Nevertheless, one may ask for those dimension vectors showing a "tame" behaviour.

A dimension vector $\mathbf{d}$ is called *tame* if there is a one parameter family of indecomposable representations for $\mathbf{d}$ and, given any (componentwise) decomposition $\mathbf{d} = \mathbf{d_1} + \mathbf{d_2}$ as a sum of two dimension vectors, all families of indecomposable representations for both $\mathbf{d_1}$ and $\mathbf{d_2}$ depend on at most one parameter.

The tame dimension vectors for trees can be characterised in completely combinatorial terms:

Theorem. Let $\mathbf{d}$ be a dimension vector of a tree Q and q denote the Tits form for Q. Then $\mathbf{d}$ is tame if and only if $q(\mathbf{d}) = 0$ and for any decomposition $\mathbf{d} = \mathbf{d_1} + \mathbf{d_2}$ as a sum of two dimension vectors, both $q(\mathbf{d_1}) \geq 0$ and $q(\mathbf{d_2}) \geq 0$.

But not only a combinatorial characterisation of the dimension vectors is possible: It can be shown (and is actually needed in the proof of the theorem above) that any tame dimension vector can be cut into "building blocks which can be described explicitly. In order to construct the families of indecomposable representations for the tame dimension vectors it is enough to give a constructive procedure for indecomposable representations of the corresponding building blocks.

There are two different kinds of building blocks for the tame dimension vectors: Taking into account *all* trees, there are only finitely many building blocks showing again a tame behaviour, and there are building blocks for which only one indecomposable representation exists. For the former ones, it is easy to construct the families of indecomposable representations. Although there are infinitely many of the latter ones, they can again be entirely described, and the corresponding indecomposable representations can be constructed explicitly by means of BGP reflection functors from simple representations.

References

[] None

Cluster structures from maximal rigid objects in 2-CY categories

Dagfinn Vatne
NULL
NTNU Trondheim
Institutt for matematiske fag, NTNU
Norway

E-mail address: **dvatne@math.ntnu.no**

For a Hom-finite 2-CY triangulated category $\mathcal{C}$, we generalise the notion of *cluster structures* from [BIRSc] to include situations where the quivers of the clusters may have loops.

An object in $\mathcal{C}$ is said to be maximal rigid if $\mathrm{Ext}^1(T,T) = 0$ and whenever $\mathrm{Ext}^1(T \oplus X, T \oplus X) = 0$, we have that X lies in $\operatorname{add} T$. We show that the set of maximal rigid objects in $\mathcal{C}$ forms a generalised cluster structure. This is a generalisation of a result in [BIRSc].

As an example, we will show that the set of maximal rigid objects in the cluster category of a tube (none of which are cluster-tilted) forms a generalised cluster structure of type B.

This is joint work with Aslak Bakke Buan and Robert Marsh.

References

[BIRSc] Buan A B, Iyama O, Reiten I, Scott J, *Cluster structures for 2-Calabi-Yau categories and unipotent groups*, preprint v. 3 arXiv:math/0701557v3 (2007)

An analysis of surface singularities by representation theory

Helmut Lenzing
Institut fuer Mathematik
Universitt Paderborn
Warburger Str. 100
Germany

E-mail address: **helmut@math.uni-paderborn.de**

Let k be an algebraically closed field. We investigate isolated quasihomogeneous surface singularities R through the quotient category $\mathcal{H}$ of all finitely generated graded R-modules by its Serre subcategory of all finitely dimensional graded R-modules. The category $\mathcal{H}$ is a hereditary noetherian category with Serre duality which, invoking a result of Reiten-van den Bergh, has an interpretation as the category of coherent sheaves on a weighted projective curve C whose underlying ordinary curve is smooth projective. We show among others that — up to a natural equivalence — R can be recovered from $\mathcal{H}$.

The central topic of the talk is the investigation of the stable derived category of finitely generated graded R-modules in the sense of R. Buchweitz, or in different terminology the triangulated category of graded singularities of R in the sense of Bondal and Orlov. We show that this category is equivalent to the stable category of vector bundles on C — which arises from a natural Frobenius category structure on the category of vector bundles on C — and investigate its structure, in particular determine its Auslander-Reiten components. Our results are particularly complete in case the ordinary curve underlying C has genus zero. In this case the triangulated categories mentioned above have tilting objects whose endomorphism rings form interesting classes of finite dimensional algebras.

Most of the research is joint work with D. Kussin, H. Meltzer and J. A. de la Peña.

References

[] None

On the Bound Quiver of a Split Extension

Sonia Trepode
NULL
Universidad Nacional de Mar del Plata
Funes 3350
Argentina

E-mail address: **strepode@mdp.edu.ar**

Joint work with I. Assem and F. Coelho. In this talk, we give a sufficient (which is also necessary under a compatibility hypothesis) condition on a set of arrows in the quiver of an algebra A so that A is a split extension of A/M, where M is the ideal of A generated by the classes of these arrows and A and A/M have compatible presentations. We apply these results to cluster tilted algebras.

References

[] None

Twisted sl2 Categorification for Spin Representations

Malka Schaps
NULL
NULL
NULL
-

E-mail address: **mschaps@macs.biu.ac.il**

The Chuang-Rouquier theory of sl2 categorification begins with an infinite family of blocks whose total Grothendieck group, after complexification, can be viewed as a highest weight representation for an affine Lie algebra of type A. The idea of the categorification is to lift the linear transformations ei and fi of the Lie algebra to functors Ei and Fi of module categories of the blocks. The theory was applied successfully to group algebras over a field of characteristic p for the symmetric groups and general linear groups, with the blocks arrayed along i-strings which are reflected by derived equivalences. In order to deal with spin representations of the symmetric groups, two changes are needed. First of all, one must consider twisted sl2 categorification, using a twisted affine Lie algebra of type A. Secondly, we conjecture that one must consider simultaneously blocks of the covering groups of Sn and of the alternating groups An, with crossovers between the two sets of blocks whenever the underlying partitions have a change of parity. As evidence for this Crossover Conjecture (Kessar-Schaps), we demonstrate that there are Morita equivalences between the extremal blocks of the i-strings, provided crossovers are taken into account. Joint with Ruthi Leabovich.

References

[] None

Finite type Artin Groups via Quiver Representations

Hugh Thomas
Department of Mathematics and Statistics
University of New Brunswick
PO Box 4400, Fredericton NB
Canada

E-mail address: **hthomas@unb.ca**

David Bessis [1] defined a certain "dual" presentation of Artin groups of finite type (that is to say, Artin groups whose corresponding Coxeter group is finite), generalizing the Birman-Ko-Lee presentation of braid groups [2]. The proof of correctness of the presentation involved type-by-type arguments and a computer check for the exceptional types. I will present a uniform proof (for crystallographic types only) based on exceptional sequences (as in Crawley-Boevey [3]) of representations of Dynkin quivers. I will also discuss the natural conjectural generalization: a presentation for arbitrary Artin groups.

References

[1] David Bessis. The dual braid monoid. Ann. Sci. École Norm. Sup. (4) 36 (2003), no. 5, 647–683.

[2] Joan Birman, Ki Hyoung Ko, and Sang Jin Lee. A new approach to the word and conjugacy problems in the braid groups. Adv. Math. 139 (1998), no. 2, 322–353.

[3] William Crawley-Boevey. Exceptional sequences of representations of quivers. Representations of algebras (Ottawa, ON, 1992), 117–124, CMS Conf. Proc. 14, Amer. Math. Soc., Providence, RI, 1993.

Composite of irreducible morphisms

Flávio Ulhoa Coelho
Matemática
Universidade de São Paulo
Rua do Matão, 1010 - Cidade Universitária
Brazil

E-mail address: **fucoelho@ime.usp.br**

It is a well-established fact that the composite of n irreducible morphisms can lie in the $(n+1)$-th power of the radical and still be non-zero. In our talk, we shall discuss this situation, surveying old and new results.

References

[] None

On the derived equivalence of the descrete categories of koszul and its yoneda algebras

Regina Aquino
NULL
UFES
Av. Fernando Ferrari,514
Brazil

E-mail address: **aquino.ufes@gmail.com**

We study the derived equivalenve between a koszul algebra and its yoneda algebra. We shall present a complete classification for the case of descrete derived categories of the koszul and its yoneda algebras. We also have described that equivalence for the simply connected koszul algebras and koszul algebras that satisfy the clock condition.

References

[] None

Finite complexity and Auslander-Reiten quivers of selfinjective algebras

Dan Zacharia
NULL
Syracuse University
Department of Mathematics
United States

E-mail address: **zacharia@syr.edu**

Let R be an artin algebra, and M an indecomposable nonprojective R-module. If

$$\cdots \to P^2 \overset{\delta_2}{\to} P^1 \overset{\delta_1}{\to} P^0 \overset{\delta_0}{\to} M \to 0$$

is a minimal projective resolution of M, then the *i-th Betti number* of M, $\beta_i(M)$, is defined to be the number of indecomposable summands of P^i. We say that the *complexity* of a finitely generated R-module M is at most n, and we write

$$cx\ M \leq n$$

if $\beta_i(M) \leq ci^{n-1}$, for some $c \in \mathbb{Q}$ and i sufficiently large and that the complexity of M is n, $cx\ M = n$, if $cxM \leq n$ but $cx\ M \not\leq n-1$. We also say that the complexity of M is infinite, if no n exists such that $cxM \leq n$. For example, $cx\ M = 0$ is equivalent to the projective dimension of M being finite, and $cx\ M = 1$ is equivalent to M having infinite projective dimension and the existence of some $b \in \mathbb{Q}$ such that $\beta_n(M) \leq b$, for all $n \geq 0$. It is well-known that if R is a selfinjective artin algebra, then the complexity is constant on each stable component of its Auslander-Reiten quiver. The purpose of this talk is to look at modules of complexity one over a selfinjective artin algebra using the structure of the Auslander-Reiten components containing them. If time permits, I will also present some preliminary results for modules of higher complexity.

References

[] None

n-APR tilting and n-cluster tilting

Steffen Oppermann
NTNU
Norway

E-mail address: **steffen.oppermann@math.ntnu.no**

By a result of Osamu Iyama (see his talk) the module category of the Auslander algebra of linear oriented A_s contains a 2-cluster tilting object, and, more generally, there is a certain n-Auslander algebra containing an $(n+1)$-cluster tilting object. In my talk I will introduce a special kind of tilting module, called n-APR tilting module. We will see that n-APR tilting acts on n-cluster tilting objects in a similar way as classical APR-tilting acts on module categories. In particular, the existence of n-cluster tilting objects is preserved. Finally, for $n = 2$, we will see how 2-APR tilting changes the algebra in terms of quivers with relations.

This is joint work in progress with Osamu Iyama.

Error Correcting Codes in Group Rings

Gladys Chalom
NULL
IME USP
av Marcos P U Rodrigues 5100
Brazil

E-mail address: **gladyschalom@gmail.com**

Error Correcting Codes in Group Rings Joint work with Raul Ferraz, Marines Guerreiro and Cesar Polcino

Error correcting Codes and their uses are every where now a days, in the supermarket, in CD s DVDs, space ships and others. Originally the algebraic theory of error correcting codes took place in the setting of vector spaces over finite fields, but the study of linear or ciclyc codes over finite rings grow of importance, because certains non linear codes are related to ideals in some rings. In some cases this ideals can be viewed as ideals of a group ring, and some invariants can be calculated.

References

[] None

ANTISYMMETRIC ELEMENTS IN GROUP RINGS

Francisco César Polcino Milies
NULL
Universidade de São Paulo
R. Acangueruçu 301
Brazil

E-mail address: **polcino@ime.usp.br**

Let R be a commutative ring, G a group and RG its group ring. Let $\varphi : RG \to RG$ denote the R-linear extension of an involution φ defined on G. An element x in RG is said to be φ-antisymmetric if $\varphi(x) = -x$. The set of antisymmetric elements of A will be denoted by $(A)^-_\varphi$. For general algebras A with an involution φ, crucial information of the algebraic structure of A can be determined by that of $(A)^-_\varphi$ and by the φ-unitary unit group $U_\varphi(A) = \{u \in A \mid u\varphi(u) = \varphi(u)u = 1\}$.

This group has been extensively studied. For example, Smirnov and Zalesskii in [3], proved that if the Lie ring generated by the elements of the form $g + g^{-1}$ with $g \in U(A)$ is Lie nilpotent then A is Lie nilpotent. In [1] Giambruno and Polcino Milies show that if A is a finite dimensional semisimple algebra over an algebraically closed field F with $char(F) \neq 2$ then $\mathcal{U}_\varphi(A)$ satisfies a group identity if and only if $(A)^-_\varphi$ is commutative. Furthermore, if F is a nonabsolute field then $\mathcal{U}_\varphi(A)$ does not contain a free group of rank 2 if and only if $(A)^-_\varphi$ is commutative. Giambruno and Sehgal, in [2], showed that if B is a semiprime ring with involution φ, $B = 2B$ and $(B)^-_\varphi$ is Lie nilpotent then $(B)^-_\varphi$ is commutative and B satisfies a polynomial identity of degree 4.

We shall give a characterization of when the φ-antisymmetric elements of RG commute. This is joint work with O. Broche Cristo, E. Jespers and M. Ruiz.

References

[1] A. Giambruno, C.Polcino Milies, Unitary units and elements in group algebras, *Manuscripta Math.* **111** (2003), no. 2, 195–209.

[2] A. Giambruno, S.K. Sehgal, Lie nilpotence of group rings, *Comm. Algebra* **21** (1993), 4253–4261.

[3] Zalesski, A. E.;A. E.; Smirnov, M. B. Lie algebra associated with linear group. *Comm. Algebra* **9** (1981), 20, 2075–2100.

THE TORSIONLESS MODULES FOR AN ARTIN ALGEBRAS.

Claus Michael Ringel
NULL
Universitaet
POB 100 131
Germany

E-mail address: **ringel@math.uni-bielefeld.de**

Let A be an artin algebra. An A-module M is said to be torsionless provided it can be embedded into a projective A-module. The algebra A is said to be torsionless-finite provided there are only finitely many isomorphism classes of indecomposable torsionless modules. The aim of the lecture will be to survey properties of the torsionless modules. In particular, we will consider the question which algebras are torsionless-finite. Note that the representation dimension of a torsionless-finite algebra is always bounded by 3.

References

[] None

New results on the finitistic dimension conjecture

Marcelo Lanzilotta Mernies
Universidad de la República
Montevideo
Uruguay

E-mail address: **marclan@cmat.edu.uy**

In this talk we present some new results related to the “finitistic dimension conjecture”. Almost all the results are from three joint works (the third in progress) with Octavio Mendoza (Universidad Nacional Autónoma de México) and François Huard (Bishop’s University, Québec).

It was conjectured by H. Bass, in the 60’s that the finitistic dimension ($\text{fin.dim}(\Lambda)$) of an Artin algebra has to be finite. Since then, much work has be done towards the proof of this conjecture.

Recently, K. Igusa and G. Todorov defined in the paper “On the finitistic global dimension conjecture for artin algebras”, a function $\Psi : \text{mod}\Lambda \to \mathbb{N}$ which turned out to be useful to prove that $\text{fin.dim}(\Lambda)$ is finite for some class of algebras.

In this talk, we show some new properties of the Igusa-Todorov function and we apply them to prove the finitistic dimension conjecture for a large family of algebras. Also, generalizing the Loewy length and having in mind the finitistic dimension conjecture, we propose the **infinite layer length**, a new measure of Λ-modules, which is an example of a more general definition: the **layer length** associated with a **torsion theory** $(\tau, \mathcal{F})$.

Abstracts presented for the special session on representations of Lie and Jordan algebras.

FILIPPOV SUPERALGEBRAS OF TYPES $B(m,n)$ AND $A(n,n)$

Alexander Pozhidaev
Institute of Mathematics SB RAS
Novosibirsk, Russia

E-mail address: **app@math.nsc.ru**

The notion of n-Lie (Filippov) superalgebra was introduced in [1] as a natural generalization of the notion of n-Lie (Filippov) algebra [2]. In [3], finite-dimensional commutative n-ary Leibniz algebras over a field of characteristic 0 were studied. It was shown there that there exist no simple ones. The finite-dimensional simple Filippov algebras over an algebraically closed field of characteristic 0 were classified in [4]. Note that an n-ary commutative Leibniz algebra is exactly a Filippov superalgebra with trivial even part, and a Filippov algebra is exactly a Filippov superalgebra with trivial odd part. Let G be a Lie superalgebra. We say that a Filippov superalgebra F has *type* G if its multiplication Lie superalgebra is isomorphic to G. In this talk, we discuss a description of Filippov superalgebras of types $B(m,n)$ and $A(n,n)$.

REFERENCES

[1] Yu.Daletskii, V.Kushnirevich, *Inclusion of Nambu-Takhtajan algebra in formal differential geometry structure*, Dop. NAN Ukr., 4 (1996) 12-18.

[2] V.T.Filippov, *n-Lie algebras*, Sib. Math. J., vol.26 (1985) 126-140.

[3] A.P.Pojidaev, *Solvability of the finite-dimensional commutative n-ary Leibniz algebras of characteristic* 0, Comm. Alg., vol.31 (2003) 197-215.

[4] Ling Wuxue, *On the structure of n-Lie algebras*, Thesis, Siegen Univ.-GHS-Siegen, (1993) 1-61.

INDECOMPOSABLE WEIGHT MODULES FOR GENERALIZED WEYL ALGEBRAS

Viktor Bekkert
Departamento de Matemtica
Universidade Federal de Minas Gerais
Av. Antnio Carlos, 6627/ Caixa Postal 702
Brazil

E-mail address: **bekkert@mat.ufmg.br**

We study the representation type of the blocks of locally-finite weight module categories for generalized Weyl algebras of degree n over an arbitrary field and describe indecomposable modules in tame blocks.

This is a joint work with Georgia Benkart (University of Wisconsin, USA) and Vyacheslav Futorny (Universidade de São Paulo, Brasil).

References

[] None

On Minimal Affinizations of Quantum Affine Algebras

Adriano Moura
NULL
UNICAMP
IMECC - UNICAMP
Brazil

E-mail address: **aamoura@ime.unicamp.br**

Let $\mathfrak{g}$ be a finite-dimensional simple Lie algebra over the field of complex numbers. Consider its loop algebra $\tilde{\mathfrak{g}}$ and the corresponding quantized enveloping algebras $U_q(\mathfrak{g})$ and $U_q(\tilde{\mathfrak{g}})$. It is known that, unless $\mathfrak{g}$ is of type A, there is no quantum group analogue of the evaluation maps $\mathfrak{g} \to \tilde{\mathfrak{g}}$. In particular, the concept of evaluation representations is not available in the context of the quantum affine algebra $U_q(\tilde{\mathfrak{g}})$. Chari and Pressley introduced and studied the concept of minimal affinizations (nowadays also called generalized Kirillov-Reshetikhin modules) which, in some sense, place the role of evaluation modules. In this talk we will discuss a few results on minimal affinizations and Kirillov-Reshetikhin modules.

References

[] None

Realizations Of An Elliptic Affine Lie Algebra

André Gimenez Bueno
Mathematics Department
Universidade de São Paulo
Rua do Matão, 1010
Brazil

E-mail address: **angbk@ime.usp.br**

This is a work done in collaboration with B. Cox and V. Futorny. In this talk we discuss how to construct boson type realizations of the elliptic affine Lie algebra $\mathfrak{sl}(2, R) \oplus \Omega_R/dR$, where $R = \mathbb{C}[t, t^{-1}, u \,|\, u^2 = t^3 - 2bt^2 + t]$.

References

[] None

$\mathbb{Z}$-forms of certain modules over Schur superalgebras

Frantisek Marko
NULL
Pennsylvania State University Hazleton
76 University drive
United States

E-mail address: **fxm13@psu.edu**

We describe the present state of the representation theory of Schur superalgebras $S = S(m|n, r)$ in positive characteristic. The category of (super)modules over a general linear supergroup is a highest weight category (proved by Zubkov) but polynomial (super)modules ((super)modules over Schur superalgebras) do not form a highest weight subcategory. Actually, in positive characteristic, a Schur superalgebra $S = S(m|n, r)$ is quasi-hereditary if and only if it is semisimple. Explicit descriptions of $S(1|1, r)$ was used by Hemmer-Kujawa-Nakano to determine the representation type of all superalgebras $S(m|n, r)$. The description of highest weights λ of simple modules over S is not straightforward and was given by Brundan-Kujawa via an algorithm related to Moullineax conjecture. Highest weights λ corresponding to $(m|n)$-hook partitions appear already in characteristic zero case. In that case the corresponding simple modules D_λ were described using (super)bideterminants by Muir. We consider its $\mathbb{Z}$-form and suitably defined (super)bideterminants. An interesting feature of these (super)bideterminants is that they are $\mathbb{Z}$-linear combinations of (super)bideterminants corresponding to semistandard tableaux.

References

[] None

Jordan Superalgebras.

Efim Zelmanov
NULL
University of California - San Diego
13333 Landfair Road
United States

E-mail address: **ezelmano@math.ucsd.edu**

We will discuss the basic examples, classification and representation theory of Jordan superalgebras. Of special interest is the small rank case, which is related to the so called superconformal algebras.

References

[] None

Representation of Jordan algebras of bilinear form

Iryna Kashuba
NULL
USP
IME-USP
Brazil

E-mail address: **ikashuba@gmail.com**

This a joint work with V. Serganova and I. Shestakov. The talk is devoted to the problem of the classication of indecomposable Jordan bimodules over nite dimensional Jordan algebras defined by bilinear form. Recall, that for a Jordan algebra J the category J-bimod of l-nite dimensional J-bimodules is equivalent to the category U-mod of (left) nitely dimensional modules over an associative algebra U = U(J), which is called the universal multiplication envelope of J. If J has nite dimension the algebra U is nite dimensional as well. It allows us to apply to the category J-bimod all the machinery developed in the representation theory of nite dimensional algebras. In particular, in accordance with the representation type of the algebra U one can dene Jordan algebras of the nite, tame and wild representation types. We classify half-unital (or one-sided) representation type for Jordan algebras of bilinear form with radical square zero.

References

[] None

On the Classification of the 1D N-Extended Superalgebra

Francesco Toppan
TEO
CBPF
Rua Dr. Xavier Sigaud 150
Brazil

E-mail address: **toppan@cbpf.br**

Several recent results concerning the classification of minimal representations of the 1D N-Extended superalgebra (the algebra of the Supersymmetric Quantum Mechanics) are presented. They include the admissible field-content of the minimal representations, the presentation of supersymmetry transformations as N-colored oriented graphs and the classification of the admissible connectivities of the associated graphs. Open problems in the representation theory of the 1D N-extended superalgebra are discussed. The talk is based on the works 1-4 (see below).

References

[1] A. Pashnev and F. Toppan, *On the Classification of the N-Extended Supersymmetric Quantum Mechanical Systems*, J. Math. Phys. vol. 42 (2001) 5257-5271.

[2] Z. Kuznetsova, M. Rojas and F. Toppan, *Classification of irreps and invariants of the N-Extended Supersymmetric Quantum Mechanics*, JHEP 0603 (2006) 098.

[3] Z. Kuznetsova and F. Toppan *Refining the classification of the irreps of the 1D N-Extended Supersymmetry*, Mod. Phys. Lett. A, vol. 23 (2008) 37-51.

[4] Z. Kuznetsova and F. Toppan, *Decomposition and Oxidation of the N-Extended Supersymmetric Quantum Mechanics Multiplets*, arXiv:0712.3176[hep-th], to be published in Int. J. Mod. Phys. A.