WAVELET SCALOGRAMS AND THEIR APPLICATIONS IN ECONOMIC TIME SERIES

[1]Miguel A. Ariño , [2]Pedro A. Morettin and [3]Brani Vidakovic

[1]University of Navarra, [2] University of São Paulo and [3] Georgia Institute of Technology

Summary
The scalogram is the discrete wavelet transformation (DWT) analogue of the well-known periodogram from the spectral analysis of time series. Just as the periodogram produces an ANOVA decomposition of the energy of a signal into different Fourier frequencies, the scalogram decomposes the energy into "level components." In this paper we show how the DWT and the scalogram can be used to detect and separate periodic components in time series. The proposed method is then used to analyse a Spanish concrete production data set.

Key words: Discrete wavelet transformations, Economic time series, Scalograms.

1 Introduction

The purpose of this paper is to implement a wavelet methodology for detecting long term trends and seasonal components in time series.

The traditional way of decomposing a (continuous or discrete) time series into components has been through spectral analysis. Suppose we have a sample fragment, $\{x_0, \ldots, x_{n-1}\}$, of a time series, and we assume that there is either zero sample mean or that the mean has been removed before analysis. From Parseval's theorem for the discrete Fourier transform it follows that the sample variance of $\{x_t\}$ is

$$s^2 = \frac{1}{n}\sum_{t=0}^{n-1} x_t^2 = \frac{1}{n}\sum_{j=0}^{n-1} |d(\omega_j)|^2, \tag{1}$$

where

$$d(\omega) = \sum_{t=0}^{n-1} x_t e^{-i\omega t} \tag{2}$$

is the discrete Fourier transform of $\{x_0, \ldots, x_{n-1}\}$ and $\omega_j = \frac{2\pi j}{n}$, $j = 0, 1, \ldots, [n/2]$ are the Fourier frequencies. Throughout this paper $[x]$ denotes the integer part of x.

Identity (1) means that the variance of the series can be decomposed into contributions given by a set of frequencies. The expression $\frac{1}{n}|d(2\pi j/n)|^2$, as a function of ω_j, is the *periodogram* of the series. The periodogram is an estimator of the true spectrum $f(\omega)$ of the process, and this gives an alternative way of looking at the series in the frequency domain instead of the time domain. If the spectrum of a (continuous or discrete) time series peaks at the frequency ω_0, then it could be concluded that in its Fourier decomposition the component with the frequency ω_0 accounts for a large part of the variance of the series. Some references for the Fourier analysis of a time series are Bloomfield (1976), Janacek and Swift (1993), and Priestley (1981).

Fourier analysis remains an important mathematical tool in many fields of science and technology. However, Fourier analysis and the decomposition of a function into simple harmonics of the form $Ae^{in\omega}$ has some drawbacks. For instance, to evaluate

$$f(\omega_0) = \int_{-\infty}^{\infty} e^{-i\omega_0\tau}\gamma(\tau)\, d\tau,$$

the spectrum f at a point ω_0 where $\gamma(\tau)$ is the autocovariance function of the series, one needs to use the function $\gamma(\tau)$ along its entire range. A change in only a few of the values of $\gamma(\tau)$ will result in a change in the value of f. In addition, the Fourier representation of local events related to a series requires many terms of the form $Ae^{in\omega}$. Various methods for time-localizing a Fourier transform have been proposed.

Lately, a new technique has arisen that has received the attention of many scientists and has rapidly diffused through the specialized literature. This is the wavelet theory and the next section contains a brief introduction to wavelets and to the discrete wavelet transform.

When analyzing a time series a classic problem has been its decomposition into components: the trend, the seasonal component, and the irregular term. Since this decomposition is not unique, and the components are interrelated, this identification is anything but simple. Several methods have been proposed to extract the components in a time series, ranging from simple weighted averages to more sophisticated methods concerning Fourier transform and spectral analysis. For example, see Kendall (1976), Bloomfield (1976), West and Harrison (1989), Brockwell and Davis (1991), and West (1995b). In economic time series, the seasonal component has usually a constant period of 12 months and to assess it one uses some underlying assumptions or theory about the nature of the series. Longer-term trends are more difficult to find (see Assimakopoulos (1995) for a recent work in long-term trend identification). By long-term trend we mean fluctuations of a time series on time scales of more than one year. Such a component is found by elimination of the seasonal component and the irregular term. Since this component is usually related to general economic fluctuations, it is often referred to as a "business cycle". West (1995a) uses Bayesian simulation analysis of autoregressions and state-space autoregressions to explore latent cyclical components in time series. See also West (1995b).

Forecasting is one of the reasons for decomposing a series. Frequently, it can be easier to forecast components of a time series than the whole series itself. For a comprehensive exposition of trends and long-term forecasting see Armstrong (1985).

In this paper we propose a methodology to estimate the long-term trend, seasonal component and irregular term of a time series.

This paper is organized as follows: after this introduction, a brief overview of wavelets is given in Section 2. This section also introduces the methodology through a simulated series. The theoretical foundations of our methodology are developed in Section 3. In Section 4, the methodology is applied to a time series of concrete production in Spain. Finally we present our conclusions in Section 5.

2 Wavelets

In this section we give a brief overview of wavelets, discrete wavelet transforms (DWT), scalograms and thresholding methods.

Wavelets are becoming increasingly popular in different areas of applied and theoretical science. Data compression, signal processing, turbulence, geophysics, statistics and numerical analysis are only a few examples from a long list of disciplines in which wavelets have been successfully employed. As far as we know, wavelet methodology has not been used extensively in economic time series studies.

Wavelets are the building blocks of wavelet transformations in the same way that the functions e^{inx} are the building blocks of the ordinary Fourier transformation. But in contrast to sines and cosines, wavelets can be (or almost can be) supported on an arbitrarily small closed time interval. Thus, wavelets are a very powerful tool in dealing with phenomena that change rapidly in time.

Statisticians are interested in wavelets as a modeling tool in the general nonlinear regression scheme. Some particular problems of interest are denoising, density and function estimation, time series, long range dependence, and change point detection.

Readers interested in mathematical and practical aspects of wavelets are directed to the monographs of Chui (1992), Daubechies (1992), and Meyer (1992). For an elementary introduction to wavelets see Strang (1993) and Vidakovic and Müller (1999). For the statistical analysis using wavelets see Ogden (1997) and Vidakovic (1999). For reviews of the uses of wavelets in statistics and time series see Morettin (1997), Antoniadis (1999), and Nason and von Sachs (1999).

Given a mother wavelet $\psi(t)$ we construct a sequence of wavelets by translations and dilations of ψ, namely

$$\psi_{j,k}(t) = 2^{-j/2}\psi(2^{-j}t - k), \quad j,k \in Z, \tag{3}$$

where $Z = \{0, \pm 1, \ldots\}$. For some choices of ψ this is an orthonormal basis. Examples are the Haar basis and the compactly supported wavelets introduced by Daubechies. Except for some special cases, there is no analytic formula for computing these wavelets. The dilation equations

$$\begin{aligned} \phi(t) &= \sqrt{2}\sum_k g_k \phi(2t-k), \\ \psi(t) &= \sqrt{2}\sum_k h_k \phi(2t-k), \end{aligned}$$

are used for this purpose, where $\phi(t)$ is the so-called scaling function or father wavelet. The coefficients g_k and h_k are low-pass and high-pass filter coefficients, respectively, satisfying $h_k = (-1)^k g_{1-k}$ (the quadrature mirror filter relation). For instance, for the $d2$ daublet from the Daubechies family, the h-filter coefficients are 0.4829629131, 0.8365163037, 0.2241438680, and -0.1294095226.

Let $\mathbf{y} = (y_0, y_1, \ldots, y_{N-1})'$ be a data vector of length $N = 2^n$. For $j = 0, 1, \ldots, n-1$ and $k = 0, 1, \ldots, 2^j - 1$ define the *discrete wavelet transform* with respect to ψ as

$$d_{j,k} = \sum_{t=0}^{N-1} y_t \psi_{j,k}(t/N). \tag{4}$$

For properties of this transform, under regularity conditions, see Chiann and Morettin (1998). If we put the $d_{j,k}$ in a vector $\mathbf{d}$, then we can write $\mathbf{d} = \mathbf{W}\mathbf{y}$. The transformation is linear and orthogonal (if appropriate boundary conditions are assumed) and can be described by an orthogonal $N \times N$ matrix $\mathbf{W}$.

In practice one finds the discrete wavelet transform (DWT) without actually exhibiting the matrix $\mathbf{W}$. This is accomplished with fast filtering algorithms based on the quadrature mirror filters referred to above which uniquely correspond to the wavelet of choice. More precisely, the wavelet decomposition of the vector $\mathbf{y}$ can be represented as the vector $\mathbf{d}$ of same size, given by

$$\mathbf{d} = (H^n\mathbf{y}, GH^{n-1}\mathbf{y}, \ldots, GH^2\mathbf{y}, GH\mathbf{y}, G\mathbf{y}) \tag{5}$$

where the operators G and H are defined via

$$(Ha)_k = \sum_n h_{n-2k} a_n, \tag{6}$$

and

$$(Ga)_k = \sum_n g_{n-2k} a_n, \tag{7}$$

where g and h are as above.

The elements of $\mathbf{d}$ are called "wavelet coefficients" (or crystals or subbands). They correspond to the levels in the tree-like indexing of wavelets in the representation of square integrable functions. For instance, the vector $G\mathbf{y}$ contains $N/2$ coefficients representing the finest detail level, the $(n-1)$st level. They are $(d_{n-1,0}, d_{n-1,1}, \ldots, d_{n-1,N/2-1})$.

The vectors $GH^{n-1}\mathbf{y} = \{d_{00}\}$ and $H^n\mathbf{y} = \{c_{00}\}$ contain a single coefficient each and represent the zeroth level of the wavelet and scaling functions respectively. In general, level j of the wavelet decomposition of $\mathbf{y}$ contains 2^j elements, and is represented by the vector $GH^{n-j-1}\mathbf{y} = (d_{j,0}, d_{j,1}, \ldots, d_{j,2^j-1})$.

Next we give a definition of the wavelet analogue of the periodogram from the Fourier analysis of time series.

If $\mathbf{d} = (c_{00}, d_{00}, d_{10}, d_{11}, d_{20}, \ldots, d_{n-1,2^{n-1}-1})$ is the vector of coefficients of the discrete wavelet transform of $\mathbf{y}$, the energy $E(j)$ of $\mathbf{d}$ at level j is defined as

$$E(j) = \sum_{k=0}^{2^j-1} d_{j,k}^2 \text{ for } j = 0, \ldots, n-1. \tag{8}$$

The *scalogram* of the data (or of $\mathbf{d}$) is the vector of energies

$$(c_{00}^2, E(0), E(1), \ldots, E(n-1)). \tag{9}$$

Since for a vector $\mathbf{v} = (v_1, \ldots, v_m)'$, $||v||^2 = \sum_{i=1}^m v_i^2$, the scalogram of $\mathbf{d}$ can be written as

$$(||H^n\mathbf{y}||^2, ||GH^{n-1}\mathbf{y}||^2, \ldots, ||GH^2\mathbf{y}||^2, ||GH\mathbf{y}||^2, ||G\mathbf{y}||^2). \tag{10}$$

There are two 0-levels, one corresponding to H^n and the second to GH^{n-1}.

Properties of the periodogram as well as of the scalogram are given in Chiann and Morettin (1998) for samples from a stationary process. For the case of nonstationary processes of a particular kind see von Sachs, Nason and Kroisandt (1998).

Wavelet shrinkage in statistics was introduced and explored in a sequence of papers by Donoho and Johnstone (available via ftp at `ftp.playfair.stanford.edu`). A good overview is given in Donoho et al. (1995).

Shrinking the wavelet image $\mathbf{d}$ of the original data set $\mathbf{y}$ and returning the shrunk version to the data domain by the inverse wavelet transformation constitutes the process of *wavelet shrinkage*. This results in the original data being denoised or compressed.

The simplest wavelet shrinkage technique is the so-called hard thresholding. The coordinates of $\mathbf{d}$ are replaced by 0 if they are smaller in absolute value than a fixed threshold λ.

The threshold λ is a "tuning" parameter of the wavelet shrinkage. Donoho and Johnstone propose several thresholds (*i.e., universal, SURE*), as well as several thresholding policies. Nason (1996) adjusted the well-known cross-validation method for use with wavelets. The threshold is selected by minimizing a cross-validatory estimator of integrated square error (ISE). A few other references in threshold selection and wavelet shrinkage applications are Gao (1997) and Vidakovic (1998, 1999).

To illustrate our methodology we consider an artificial time series which is the sum of two perfect periodic series. Looking at the scalogram of the corresponding wavelet decomposition we will be able to split the wavelet decomposition into two parts. We shall then recover the two components using the scalogram.

Figure 1 (a) shows two discrete, equally spaced series, $\mathbf{y} = \{y_t\}$ and $\mathbf{z} = \{z_t\}$, obtained from two periodic functions, namely, a sinusoidal function with four waves and a function like a saw with 32 teeth. Figure 1 (b) shows its sum $x = \{x_t\}$, given by $x_t = y_t + y_t$, for $t = 1, \ldots, N$, where N is a power of two, especifically, 2^{12} in this case. Since $\{x_t\}$ is a sample from a continuous function, we should chose a wavelet basis whose elements are smooth functions. The wavelet basis chosen to decompose $\{x_t\}$ is the d8 basis whose mother wavelet function has 8 vanishing moments and whose Hölder exponent is approximately 2.

The scalogram of the discrete wavelet transform of x, as defined in (8), is presented in Figure 1 (c)[1]

The existence of a peak in the scalogram of a time series at a high level j indicates that a high-frequency component is present in the series. When there is a peak at a low level j, this means that a low-frequency component is present. In fact, frequencies are connected with levels in the decomposition via domains of wavelets

[1]The first zeroth level corresponding to c_{00}^2 is omitted in the graphic representation of the scalogram.

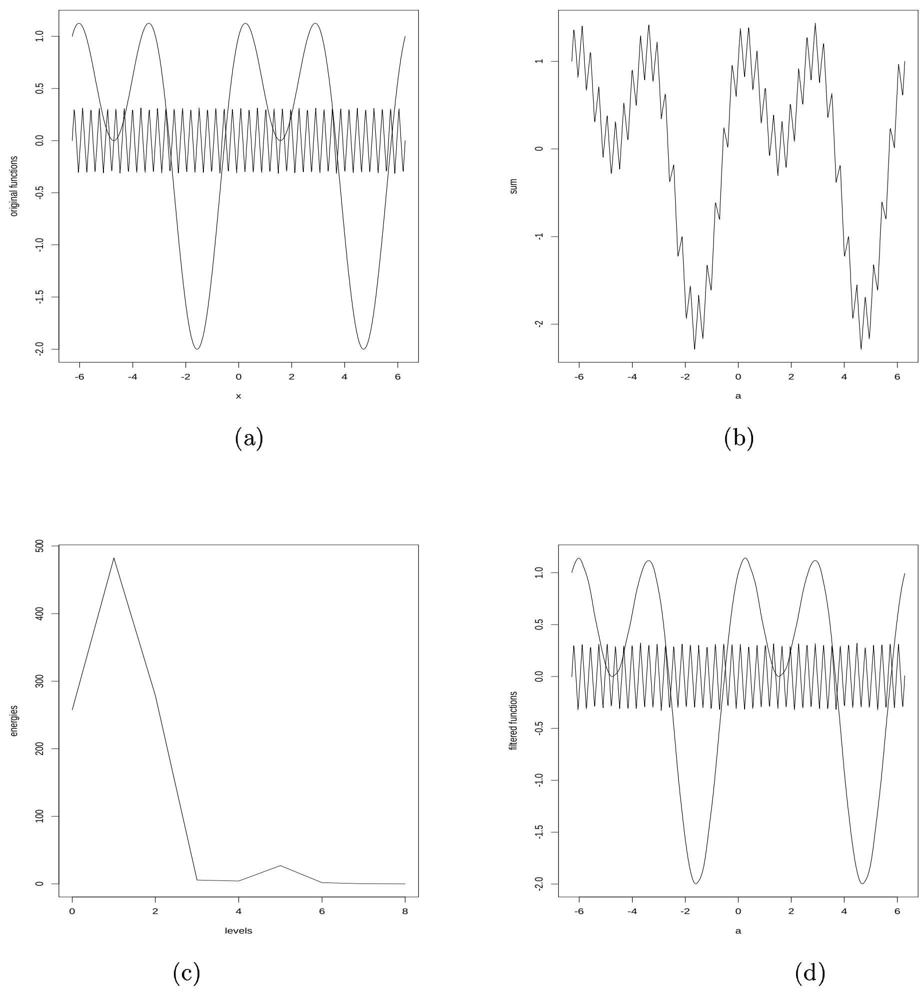


Figure 1: A two-cycle composition series (upper panel, right) and its components (upper panel, left), its scalogram (lower panel, left) and its cycle decomposition (lower panel, right).

at level j. When two or more peaks are present in the scalogram, we can identify the corresponding components by splitting the wavelet decomposition of the series in two decompositions; the first taking the coefficients around the first peak padded by zeroes, and the second taking the coefficients around the second peak also padded by zeroes. The way to analyze the levels between the peaks will be explained in Section 5.

From the scalogram presented in Figure 1 (c) we conclude that the wavelet decomposition of x can be split into two wavelet decompositions. The first has the coefficients from level 0 to level 4; the second has the coefficients from level 5 to level 11. Analytically,

$$d_{j,k}^{(1)} = \begin{cases} d_{j,k} & \text{for } j = 0, 1, \ldots, 5 \\ 0 & \text{for } j = 6, \ldots, 11 \end{cases}$$

$$d_{j,k}^{(2)} = \begin{cases} 0 & \text{for } j = 0, 1, \ldots, 5 \\ d_{j,k} & \text{for } j = 6, \ldots, 11 \end{cases}$$

$$c_{00}^{(1)} = c_{00}, \text{ and } c_{00}^{(2)} = 0.$$

These two wavelet decompositions are the discrete wavelet transformations of two data vectors $\mathbf{y}' = \{y_t'\}$ and $\mathbf{z}' = \{z_t'\}$. Specifically,

$$\mathbf{y}' = \mathbf{W}^{-1}\mathbf{d}^{(1)},$$

and

$$\mathbf{z}' = \mathbf{W}^{-1}\mathbf{d}^{(2)}.$$

In Figure 1 (d) we present the reconstructed series $\mathbf{y}'$ and $\mathbf{z}'$. We can see that both are very similar to $\mathbf{y}$ and $\mathbf{z}$ except at the ends of the interval $[0, 8\pi]$. That is due to boundary effects in the wavelet decomposition algorithm. Although the example shown here is simple, it illustrates how a time series can be decomposed into components of different frequencies.

Note that it has not been necessary to threshold here since the artificially constructed time series $\{x_t\}$ had no noise included. In general, when analyzing a real economic time series, as in the application below, we will have to threshold the series in order to remove the noise before determining the hidden components.

3 Analytical Issues

The decomposition of a time series into hidden components, hinted at in Section 2, is based on the scalogram analysis of the discrete wavelet decomposition of the time series with respect to a fixed wavelet basis. If two peaks are present in the scalogram, we propose to split the wavelet decomposition $\mathbf{d}$ of the time series $\{x_t\}$ into two new wavelet decompositions, $\mathbf{d}^{(1)}$ and $\mathbf{d}^{(2)}$, in the following way: the coefficients $d_{j,k}$ of $\mathbf{d}$ which are in levels j close to the first peak are assigned to $\mathbf{d}^{(1)}$ ($d^{(1)}_{j,k} = d_{j,k}$). The corresponding coefficients in $\mathbf{d}^{(2)}$ will be zero ($d^{(2)}_{j,k} = 0$). Similarly, the coefficients $d_{j,k}$ of $\mathbf{d}$ for levels j close to the second peak are assigned to $\mathbf{d}^{(2)}$, and the corresponding coefficients in $\mathbf{d}^{(1)}$ will be zero. A problem arises when a level occurs between the peaks because it is then unclear how to assign coefficients. We propose to split the coefficients of that level and join each part to each split wavelet decomposition.

In this section we will suggest two ways to split the coefficients of intermediate levels. The first method is additive with respect to energies but not with respect to wavelet coefficients. That is, it is not additive in the scale domain. The second method is additive with respect to wavelet coefficients but does not preserve energies.

Let j be a level such that its energy $E(j)$ is between two peaks in the scalogram. The energy-preserving method splits these coefficients as follows:

$$d^{(1)}_{j,k} = \sqrt{\frac{a}{a+b}}\, d_{j,k} \tag{11}$$

and

$$d^{(2)}_{j,k} = \sqrt{\frac{b}{a+b}}\, d_{j,k}, \tag{12}$$

where

$$a = d^2_{j-1,[k/2]}, \tag{13}$$

and

$$b = \frac{d^2_{j+1,2k} + d^2_{j+1,2k+1}}{2}. \tag{14}$$

The rationale behind this way of splitting is the following: each coefficient $d_{j,k}$ has a "generator" in its superior level $j-1$ which is $d_{j-1,[k/2]}$. Also, each $d_{j,k}$ generates the two coefficients, $d_{j+1,2k}$ and $d_{j+1,2k+1}$, in its inferior level $j+1$. The coefficient $d^{(1)}_{j,k}$ is "proportional" to the relative size of its generator with respect to the total size of its generator and the average of both coefficients that $d_{j,k}$ generates at level $j+1$.

Under this splitting we have

$$(d_{j,k})^2 = \left(d_{j,k}^{(1)}\right)^2 + \left(d_{j,k}^{(2)}\right)^2,$$

but

$$d_{j,k} \neq d_{j,k}^{(1)} + d_{j,k}^{(2)}.$$

Hence, we say this method is energy-preserving but not additive.

However, if the coefficients are split in the following way:

$$d_{j,k}^{(1)} = \frac{a}{a+b} d_{j,k} \tag{15}$$

and

$$d_{j,k}^{(2)} = \frac{b}{a+b} d_{j,k}, \tag{16}$$

where a and b are given by (13) and (14), then the method is additive but the energies are no longer preserved.

This follows immediately since

$$(d_{j,k})^2 \neq \left(d_{j,k}^{(1)}\right)^2 + \left(d_{j,k}^{(2)}\right)^2,$$

but

$$d_{j,k} = d_{j,k}^{(1)} + d_{j,k}^{(2)}.$$

In determining how to split the coefficients of a given level j, we suggest an examination of the energy of that level $E(j)$. When $E(j)$ is quite large with respect to the two peaks of the scalogram, our advice is to use the additive method of splitting. Otherwise, the components in which the time series of interest is to be decomposed would not add up to the total time series. On the contrary, if $E(j)$ is small with respect to the size of the peaks, the energy-preserving method is better to use. Even though the components do not sum the total series with the energy-preserving method, the remaining part is quite small and can be disregarded. For each particular time series the analyst has to decide which method to use.

4 An Application

In this section we introduce a real data set to which the methodology in this paper will be applied. The data set is a time series of the monthly production of concrete in Spain from January 1955 to January 1995[2]. A plot of this monthly production is given in Figure 2. The data are measured in thousands of tons.

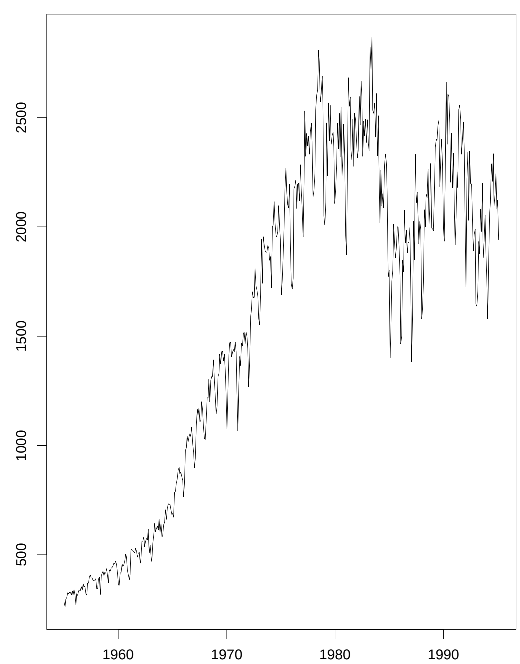


Figure 2: Monthly concrete production in Spain 1955-1994.

As shown in Figure 2, concrete production in Spain increased on the average from 1955 until the mid-seventies. In that period Spain was a rapidly developing country but has since become a developed one. In this second period concrete production exhibited larger fluctuations than the economy in general. In the early 1980's, the major economic crisis that struck the developed world in 1979 arrived in Spain and affected the construction sector in a special way. That crisis was followed by an economic boom during the late 1980's which was emphasized in Western Europe by

[2]Data extracted from Boletín Económico del Banco de España

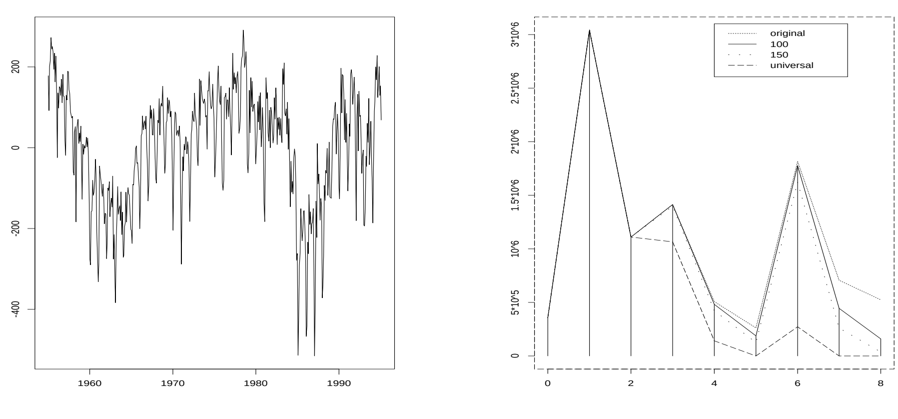
the prospect of economic unification. During that period the construction sector in Spain was also active due to the infrastructure needed to host the 1992 Barcelona Olympic Games. Concrete production in Spain was also affected by the crisis that at the begining of the 90's affected the western economies.

Within each year there is a seasonal fluctuation of concrete production: cold weather and Christmas holidays slow down the production of concrete during the winter. This is illustrated in Figure 2 by the troughs of one year periodicity. Furthermore, in the summer (mainly August), activity in Spain slows down in general and this is reflected in a decrease in production.

Having presented the methodology to extract the cycles in a time series, let us return to analyze the series $\{x_t\}$ of monthly concrete production in Spain. First, in order to stabilize the variance in $\{x_t\}$, we take natural logarithms and let $x'_t = 1000 \log x_t$. In addition, we fit a parabola to $\{x'_t\}$ to remove the trend and study the residual series

$$y_t = 1000 \log x_t - 5450.92 - 12.7917t + 0.01773t^2. \quad (17)$$

Let T be the transformation that makes $T(x_t) = y_t$. A plot of the residuals $\mathbf{y} = \{y_t\}$ is presented in Figure 3(a).



(a) (b)

Figure 3: Residual series $\{y_t\}$, and the scalograms of the DWT of $\underset{\sim}{d}$ under several thresholds.

We assume the series $\mathbf{y}$ is stationary with mean 0. In order to analyze the scalogram of the wavelet decomposition of $\{y_t\}$ we have chosen to decompose the series with the *d8* wavelet basis as we did in the example in Section 3. To decompose $\{y_t\}$ it is convenient that the data set be of a size which is a power of two. Since we have 481 data points we add at the beginning of the series 31 "zeroes", so that our time series $\mathbf{y}$ has $512 = 2^9$ data points. After obtaining the decomposition

$$\mathbf{d} = \mathbf{W}\mathbf{y}$$

we threshold the coefficients to remove the noise by replacing all coefficients smaller than 100 with 0. This threshold has been chosen so that the scalogram of the thresholded decomposition keeps the peaks it had before thresholding, almost unchanged, while the noise is reduced. Small coefficients $d_{j,k}$ at levels away from the peaks can be regarded as noise. The Donoho-Johnstone universal threshold is 358. Using that threshold would eliminate the seasonal effects of the series. The scalograms of the non-thresholded series, and those of the thresholded series with thresholds 100, 150, and the universal, are shown in Figure 3 (b). The set of thresholded coefficients will be denoted by $\tilde{\mathbf{d}}$.

Levels 1 and 6 present two major peaks in the scalogram of $\tilde{\mathbf{d}}$. As discussed in Section 3, we decompose $\tilde{\mathbf{d}}$ into $\tilde{\mathbf{d}}^{(1)}$ and $\tilde{\mathbf{d}}^{(2)}$, each also of size 2^9; $\tilde{\mathbf{d}}^{(1)}$ contains the coefficients of levels 0 to 4 of $\tilde{\mathbf{d}}$ and $\tilde{\mathbf{d}}^{(2)}$ contains the coefficients of levels 6 to 8 of $\tilde{\mathbf{d}}$. The coefficients of level 5 are split according to the energy-preserving algorithm given in the previous section. Had we chosen the additive method of splitting, the difference would have been minor since the energy is small. The remaining coefficients of $\tilde{\mathbf{d}}^{(1)}$ and $\tilde{\mathbf{d}}^{(2)}$ are 0.

In order to extract the cycles in $\mathbf{y}$ we apply the inverse discrete wavelet transformation to $\tilde{\mathbf{d}}^{(1)}$ and $\tilde{\mathbf{d}}^{(2)}$. We obtain

$$\mathbf{y}^{(1)} = \mathbf{W}^{-1}\tilde{\mathbf{d}}^{(1)}, \tag{18}$$

and

$$\mathbf{y}^{(2)} = \mathbf{W}^{-1}\tilde{\mathbf{d}}^{(2)}. \tag{19}$$

Our last step is to extract the components in the original series $\{x_t\}$ by using the inverse transformation T^{-1}. This series is to be decomposed as

$$\{x_t\} = \{x_t^{(1)}\} + \{x_t^{(2)}\} + \{e_t\}. \tag{20}$$

The first component[3] $x_t^{(1)}$ is

$$x_t^{(1)} = T^{-1}\left(y_t^{(1)}\right). \tag{21}$$

This series $\{x_t^{(1)}\}$ captures the business cycle that affects all industries and not only the construction sector.

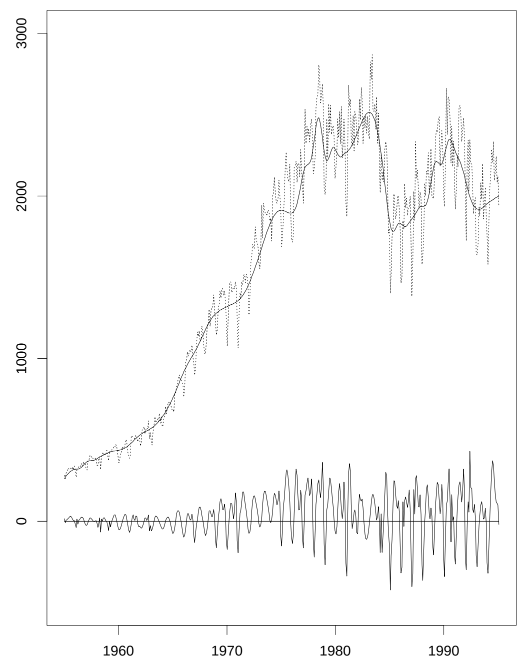


Figure 4: Original series for concrete production data set and decomposition of series into cycles.

Since $\{e_t\}$ is the noise term removed through thresholding we have

$$e_t = x_t - T^{-1}\left(\underset{\sim}{y}_t^{(1)} + \underset{\sim}{y}_t^{(2)}\right). \tag{22}$$

[3]Of course, the 31 data points added at the beginning of the series to make it of length 512 have to be removed.

Finally, the second component is the remaining part

$$x_t^{(2)} = x_t - x_t^{(1)} - e_t \tag{23}$$

which captures the seasonality of this sector, as explained in section 4.

A graph of the total series $\{x_t\}$ and the business cycle series $\{x_t^{(1)}\}$, together with the seasonal component $\{x_t^{(2)}\}$, is presented in Figure 4.

5 Conclusion

We have presented a method to extract hidden components in a time series through the use of the recently developed wavelet theory. Although decomposition of a time series has been a long-time subject of study, it is by no means a closed problem. In fact, each attempt to decompose a series calls for a specific method to be applied. The method proposed consists of splitting the wavelet coefficients, with respect to a specified basis, of the data set to be analyzed. The decision of how to split these wavelet coefficients is made after an analysis of its scalogram. Although our main purpose has been to present a methodology, it has also been applied to the economic time series of monthly Spanish concrete production. In that series we have found two components. The first is a low-frequency, long-period component that captures the general economic trend in Spain as influenced by the world economic situation. It can be considered the long-term trend of the series which changes with the pulse of the general economy. The second component has a twelve-month period and reflects the typical seasonality of the construction sector in Spain.

The methodology presented could be applied to other economic time series. If economic components are found, and they are related, then perhaps they could be used for general economic activity prediction.

The Bayesian approach can be used to choose shrinkage methods. In general, Bayes rules are *shrinkers*, are less ad hoc than other proposals and have been shown to be effective. If Bayes models for wavelet coefficients are used, the resulting optimal actions can be very close to thresholding. We decided to use a simple thresholding rule in our case and for further details on Bayesian proposals see Chipman et al. (1997) and Vidakovic (1998).

References

ANTONIADIS, A. (1999). Wavelets in statistics: A review. *Journal of the Italian Statistical Society*, **6**.

ASSIMAKOPOULOS, V. (1995). A successive filtering technique for identifying long-term trends. *Journal of Forecasting,* **14**, 35-43.

ARMSTRONG, J.S. (1985). *Long-Range Forecasting.* New York: John Wiley.

BLOOMFIELD, P. (1976). *Fourier Analysis of Time Series: An Introduction*, New York: Wiley.

BE. Boletín Económico del Banco de España, Several issues.

BROCKWELL, P. and DAVIS, R. (1991). *Time Series: Theory and Methods.* Second Edition. New York: Springer-Verlag.

CHIANN, C. and MORETTIN, P.A. (1998). A wavelet analysis for time series. *J. Nonparametric Statistics*, **10**, 1–46.

CHIPMAN, H., MCCULLOCH, R. and KOLACZYK, E. (1997). Adaptive Bayesian wavelet shrinkage. *Journal of the American Statistical Association*, **92**, 1413–1421.

CHUI, K. (1992). *An Introduction to Wavelets.* New York: Academic Press.

DAUBECHIES, I. (1992). *Ten Lectures on Wavelets*, SIAM. CBMS-NSF Conference Series **61**.

DONOHO, D., JOHNSTONE, I., KERKYACHARIAN, G., and PICARD, D. (1995). Wavelet shrinkage: Asymptopia? *J. R. Statist. Soc.*, **57 (2)**, 301-369.

GAO, H-Y. (1997). Choice of thresholds for wavelet shrinkage estimate of the spectrum. *J. Time Series Analysis,* **18**, 231–251.

JANACEK, G. and SWIFT, L. (1993). *Time Series. Forecasting, Simulation, Applications.* Ellis Horwood.

KENDALL, M.G. (1976). *Time Series.* Second Edition. London: Charles Griffin and Company, Ltd.

MEYER,Y. (1992). *Wavelets and Operators.* Cambridge: Cambridge University Press.

MORETTIN, P.A. (1997). Wavelets in statistics. *Resenhas Journal*, **3**, 211–272.

NASON, G. (1996). Wavelet shrinkage using cross-validation. *J. Royal Statist. Society*, Series B, **58**, 463–479.

NASON, G. and VON SACHS, R. (1999). Wavelets in time series analysis. *Phil. Trans. Royal Soc. London, A*, 357, 2511–2526.

NASON, G., VON SACHS, R. and KROISANDT, G. (2000). Wavelet processes and adaptive estimation of the evolutionary wavelet spectrum. *J. Royal Statist. Society*, Series B, **62**, 271–292.

OGDEN, R.T. (1997). *Essential Wavelets for Statistical Applications and Data Analysis.* Boston: Birkhäuser.

PRIESTLEY, M.B. (1981). *Spectral Analysis and Time Series.* London: Academic Press.

STRANG, G. (1993). Wavelet transforms versus Fourier transforms. *Bulletin (New Series) of the American Mathematical Society,* **28**(2), 288-305.

VIDAKOVIC, B. (1995). Unbalancing data with wavelet transformations. In *Proceedings SPIE, 2569, Wavelet Applications in Signal and Image Processing III.*

VIDAKOVIC, B. (1998). Nonlinear wavelet shrinkage with Bayes rules and Bayes factors. *Journal of the American Statistical Association*, **93,** 173–179.

VIDAKOVIC, B. (1999). *Statistical Modeling by Wavelets.* New York: Wiley.

VIDAKOVIC, B. and MÜLLER, P. (1999). An introduction to wavelets. In: *Bayesian Inference in Wavelet-Based Models.* Editors Müller and Vidakovic, Springer-Verlag, Lecture Notes in Statistics **141**, 1–18.

WEST, M. (1995a). Time series decomposition and analysis in a study of oxygen isotope records, Discussion Paper **95-18**, ISDS, Duke University.

WEST, M. (1995b). Bayesian inference in cyclical component dynamic linear models. *J. Amer. Statist. Assoc.*, **90**, 1301–1312.

WEST, M. and HARRISON, J. (1989). *Bayesian Forecasting and Dynamic Models.* New York: Springer-Verlag.

Miguel Ariño	Pedro A. Morettin	Brani Vidakovic
IESE Business School	Department of Statistics	Industrial and Systems Engineering
University of Navarra	University of São Paulo	Georgia Institute of Technology
08034 Barcelona, Spain	05311-970 São Paulo, Brazil	Atlanta, GA 30332