MAT 133 - CÁLCULO II - IQUSP
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Período: Segundo Semestre de 2013

8ª LISTA DE EXERCÍCIOS

1. Mostre que para todo $x \neq 1$ e quaisquer que sejam $n, N \in \mathbb{N}$, com $N \geq n$, temos
$$\sum_{j=n}^{N} x^j = \frac{x^n - x^{n+N+1}}{1-x} \quad .$$

2. Verifique as fórmulas abaixo.
   (a) $\sum\limits_{j=1}^{n} j = \frac{n(n+1)}{2}$.
   (b) $\sum\limits_{j=1}^{n} j^2 = \frac{n(n+1)(2n+1)}{6}$.
   (c) $\sum\limits_{j=1}^{n} j^3 = \left(\frac{n(n+1)}{2}\right)^2$.

3. Mostre que $\sum\limits_{j=1}^{n}\sum\limits_{k=1}^{m} x_j y_k = \sum\limits_{k=1}^{m}\sum\limits_{j=1}^{n} x_j y_k = \left(\sum\limits_{j=1}^{n} x_j\right)\left(\sum\limits_{k=1}^{m} y_k\right)$.

4. Sejam $(x_j)_{1\leq j\leq n}$ e $(y_k)_{1\leq k\leq n}$ duas sequências finitas em $\mathbb{C}$. Verifique
$$\left(\sum_{j=1}^{n} x_j y_j\right)^2 = \left(\sum_{j=1}^{n} x_j^2\right)\left(\sum_{k=1}^{n} y_k^2\right) - \sum_{1\leq j<k\leq n} (x_j y_k - x_k y_j)^2 \ .$$

5. Verifique a Propriedade Telescópica:
$$\sum_{k=m}^{n} (z_{k+1} - z_k) = z_{n+1} - z_m .$$

6. Calcule, aplicando a propriedade telescópica,
   (a) $\sum\limits_{k=1}^{n} [\ (k+1)^3 - k^3\ ]$.
   (b) $\sum\limits_{j=2}^{n} \frac{1}{j(j-1)}$
   (c) $\sum\limits_{j=100}^{500} \frac{1}{j(j+1)(j+2)}$

   Sugestão para (c): verique que $\frac{1}{j(j+1)(j+2)} = \frac{1}{2}\left(\frac{1}{j(j+1)} - \frac{1}{(j+1)(j+2)}\right)$

7. Calcule a soma da série dada.

(a) $\sum\limits_{k=0}^{+\infty} \left(\frac{1}{10}\right)^k$.

(b) $\sum\limits_{k=0}^{+\infty} \pi^{-k}$.

(c) $\sum\limits_{k=0}^{+\infty} \frac{1}{(4k+1)(4k+5)}$.

(d) $\sum\limits_{k=1}^{+\infty} \frac{1}{(k+1)(k+2)(k+3)}$.

8. Calcule a soma da série dada

(a) $\sum\limits_{n=1}^{+\infty} \frac{1}{n(n+1)(n+2)(n+3)}$

(b) $\sum\limits_{n=1}^{+\infty} n\alpha^n$, $0 < \alpha < 1$.

9. Determine a convergência ou divergência das séries (v. Guidorizzi, Vol. 4).

(a) $\sum\limits_{k=0}^{+\infty} \frac{1}{k^2+1}$.

(b) $\sum\limits_{k=3}^{+\infty} \frac{1}{k^2 \log(k)}$.

(c) $\sum\limits_{k=0}^{+\infty} \frac{\sqrt{k}}{1+k^4}$

(d) $\sum\limits_{p=4}^{+\infty} \log \frac{2p}{p+1}$

(e) $\sum\limits_{n=5}^{+\infty} \frac{n^2-3n+1}{n^2+4}$.

10. Determine se convergem ou não as séries abaixo.

(a) $\sum\limits_{k=2}^{+\infty} \frac{k}{4k^3-k+10}$.

(b) $\sum\limits_{k=2}^{+\infty} \frac{(k+1)e^{-k}}{2k+3}$.

(c) $\sum\limits_{k=2}^{+\infty} \frac{\sqrt{k}+\sqrt[3]{k}}{k^2+7k+11}$.

(d) $\sum\limits_{k=20}^{+\infty} \frac{2^k}{k^5}$.

(e) $\sum\limits_{k=1}^{+\infty} \frac{2^k}{k!}$

(f) $\sum\limits_{k=3}^{+\infty} \frac{1}{k(\log k)^{10}}$

(g) $\sum\limits_{n=2}^{+\infty} \frac{1}{n\sqrt[3]{n^2+3n+1}}$.

11. Determine se convergem ou não as séries abaixo.

(a) $\sum\limits_{n=0}^{+\infty} \frac{3^n}{1+4^n}$.

(b) $\sum\limits_{n=1}^{+\infty} \frac{n!\,2^n}{n^n}$.

(c) $\sum\limits_{n=3}^{+\infty} [\sqrt{n+1} - \sqrt{n}]$.

(d) $\sum\limits_{n=4}^{+\infty} \frac{n^3+4}{2^n}$

12. Estude, com relação à convergência ou divergência:

(a) $\sum\limits_{k=0}^{+\infty} \frac{k}{k^2+1}$

(b) $\sum\limits_{n=1}^{+\infty} \frac{n}{\sqrt[n]{n}}$.

13* Determine os valores de $\alpha \geq 0$ e $\beta \geq 0$ tais que são convergentes as séries:

(a) $\sum\limits_{n=2}^{+\infty} \frac{1}{n^\alpha (\log n)^\beta}$

(b) $\sum\limits_{n=2}^{+\infty} \frac{(\log n)^\beta}{n^\alpha}$.

14* Seja $\alpha \in \mathbb{R} \setminus \mathbb{N}$. Consideremos a sequência $(|a_n|)$, $n \geq 1$, dos coeficientes binomiais $a_n = \binom{\alpha}{n} = \frac{\alpha(\alpha-1)(\alpha-2).....(\alpha-n+1)}{n!}$. Verifique as afirmações abaixo.

(a) Se $-1 < \alpha$ então $\lim a_n = 0$ e $(|a_n|)_{n \geq n_0}$, $n_0 > \alpha$, decresce.

(b) Se $\alpha < -1$, α inteiro ou não, então $\lim a_n \neq 0$.

(c) Se $\alpha < -1$ então $\sum\limits_{n=0}^{+\infty} \binom{\alpha}{n}$ diverge.

15* Seja $0 < \alpha < 1$. Então,

(a) A série (não alternada) $\sum\limits_{n=1}^{+\infty} (-1)^{n+1} \frac{\alpha(\alpha-1)\cdots(\alpha-n+1)}{n!}$ é convergente.

(b) A série $\sum\limits_{n=1}^{+\infty} \frac{\alpha(\alpha-1)\cdots(\alpha-n+1)}{n!}$ é alternada e convergente.

16* Se $-1 < \alpha < 0$ então $\sum\limits_{n=0}^{+\infty} \binom{\alpha}{n}$ converge condicionalmente.

17* Mostre que $\sum\limits_{n=0}^{+\infty} \binom{\alpha}{n} x^n$, $x \in \mathbb{R}$, com $\alpha \in \mathbb{R} \setminus \mathbb{N}$, satisfaz,

(a) Diverge, se $|x| > 1$, qualquer que seja $\alpha \in \mathbb{R} \setminus \mathbb{N}$.

(b) Converge absolutamente, se $|x| < 1$, qualquer que seja $\alpha \in \mathbb{R} \setminus \mathbb{N}$.

(c) Se $\alpha > 0$, converge (absolutamente) se somente se $x \in [-1, 1]$.

(d) Se $-1 < \alpha < 0$, converge se somente se $x \in (-1, 1]$ e converge condicionalmente se $x = 1$.

(e) Se $\alpha \leq -1$, converge se e somente se $x \in (-1, 1)$.

18. A série $\sum\limits_{n=1}^{+\infty} \frac{1\cdot3\cdot5\cdots(2n-1)}{2\cdot4\cdot6\cdots2n}$ é convergente ou divergente? Justifique.

19. Determine se é convergente ou divergente a série dada abaixo.

(a) $\sum\limits_{n=1}^{+\infty} \frac{\cos^2 n + n^2}{n^4}$

(b) $\sum\limits_{n=1}^{+\infty} n^2 (1 - \cos \frac{1}{n^2})$

(c) $\sum\limits_{n=1}^{+\infty} \log(1 + \frac{1}{n^2})$

(d) $\sum\limits_{n=3}^{+\infty} \frac{\sqrt[3]{n^5+3n+1}}{n^3 (\log n)^2}$

(e) $\sum\limits_{n=3}^{+\infty} \frac{(\log n)^3}{n^2}$

(f) $\sum\limits_{n=1}^{+\infty} \arctan\left(\frac{1}{n\sqrt[3]{n^2+3}}\right)$

(g) $\sum\limits_{n=1}^{+\infty} \left(\frac{n^2+5}{n^2+3} - 1\right)$

(h) $\sum\limits_{n=1}^{+\infty} \log\left(\frac{n^2+5}{n^2+3}\right)$.

20. Determine se é convergente ou divergente a série dada abaixo.

(a) $\sum\limits_{n=0}^{+\infty} \frac{2^n}{1+3^n}$

(b) $\sum\limits_{n=1}^{+\infty} \frac{1}{2^n} (1 + \frac{1}{n})^n$

(c) $\sum\limits_{n=0}^{+\infty} \frac{n^2}{n!}$

(d) $\sum\limits_{n=1}^{+\infty} \frac{n!}{n^n}$

(e) $\sum\limits_{n=1}^{+\infty} 3^n \frac{n!}{n^n}$

(f) $\sum\limits_{n=1}^{+\infty} \frac{n!}{3.5.7.....(2n+1)}$

(g) $\sum\limits_{n=1}^{+\infty} \frac{2.4.6.....(2n)}{n^n}$.