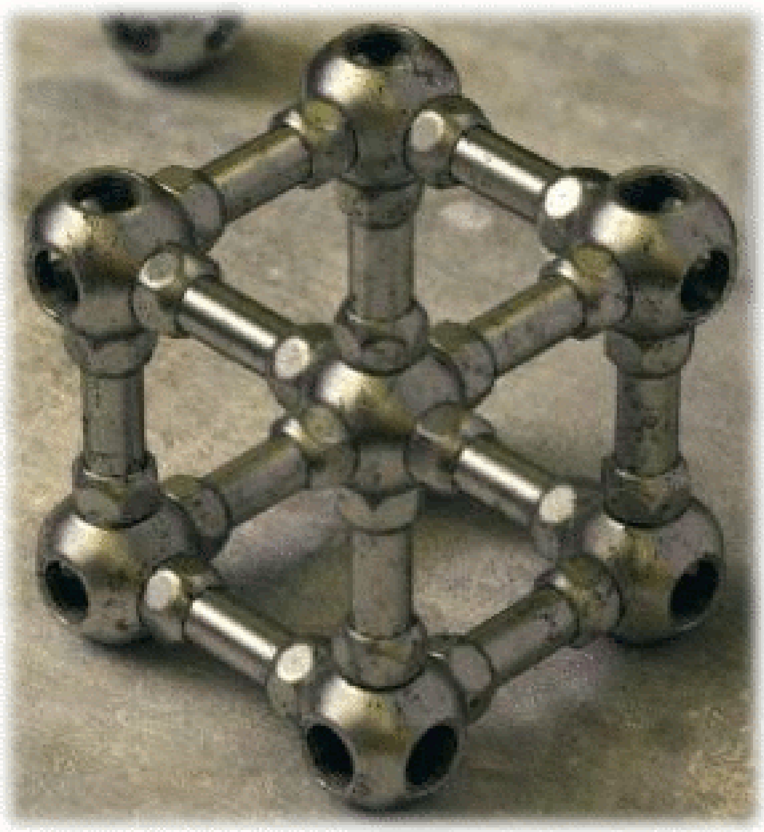

Topics in Horn Contraction: Supplementary Postulates, Package Contraction, and Forgetting [1]

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Topics in Horn Contraction: Supplementary Postulates, Package Contraction, and Forgetting

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Abstract

In recent years there has been interest in studying belief change, specifically contraction, in Horn knowledge bases. Such work is arguably interesting since Horn clauses have found widespread use in AI; as well, since Horn reasoning is weaker than classical reasoning, this work also sheds light on the foundations of belief change. In this paper, we continue our previous work along this line. Our earlier work focussed on defining contraction in terms of *weak remainder sets*, or maximal subsets of an agent's belief set that fail to imply a given formula. In this paper, we first examine issues regarding the *extended contraction postulates* with respect to Horn contraction. Second, we examine *package contraction*, or contraction by a set of formulas. Last, we consider the closely-related notion of *forgetting* in Horn clauses. This paper then serves to address remaining major issues concerning Horn contraction based on remainder sets.

1 Introduction

Belief change addresses how a rational agent may alter its beliefs in the presence of new information. The best-known approach in this area is the AGM paradigm [Alchourrón *et al.*, 1985; Gärdenfors, 1988], named after the original developers. This work focussed on belief *contraction*, in which an agent may reduce its stock of beliefs, and belief *revision*, in which new information is consistently incorporated into its belief corpus. In this paper we continue work in belief contraction in the expressively weaker language of *Horn formulas*, where a Horn formula is a conjunction of Horn clauses and a Horn clause can be written as a rule in the form $a_1 \wedge a_2 \wedge \cdots \wedge a_n \rightarrow a$ for $n \geq 0$, and where a, a_i $(1 \leq i \leq n)$ are atoms. (Thus, expressed in conjunctive normal form, a Horn clause will have at most one positive literal.) Horn contraction has been addressed previously in [Delgrande, 2008; Booth *et al.*, 2009; Delgrande and Wassermann, 2010; Zhuang and Pagnucco, 2010b]. With the exception of the last reference, this work centres on the notion of a *remainder set*, or maximal subset of a knowledge base that fails to imply a given formula.

In this paper we continue work in Horn belief contraction, on a number of aspects; our goal is to essentially complete the overall framework of Horn contraction based on remainder sets. Previous work in this area has addressed counterparts to the *basic* AGM postulates; consequently we first examine prospects for extending the approach to counterparts of the *supplemental* AGM postulates. Second, we address *package contraction*, in which one may contract by a set of formulas, and the result is that no (contingent) formula in the set is believed. In the AGM approach, for a finite number of formulas this can be accomplished by contracting by the disjunction of the formulas. Since the disjunction of Horn formulas may not be in Horn form, package contraction then becomes an important accessory operation. Last we briefly examine a *forgetting* operator, in which one effectively reduces the language of discourse.

The next section introduces belief change while the third section discusses Horn clause reasoning, and previous work in the area. Section 4 examines the supplementary postulates; Section 5 addresses package contraction; and Section 6 covers forgetting. The last section contains a brief conclusion.

2 The AGM Framework for Contraction

As mentioned, the AGM approach [Alchourrón *et al.*, 1985; Gärdenfors, 1988] is the best-known approach to belief change. Belief states are modelled by deductively-closed sets of sentences, called *belief sets*, where the underlying logic includes classical propositional logic. Thus a belief set K satisfies the constraint:

$$\text{If } K \text{ logically entails } \phi \text{ then } \phi \in K.$$

The most basic operator is called *expansion*: For belief set K and formula ϕ, the expansion of K by ϕ, $K + \phi$, is the deductive closure of $K \cup \{\phi\}$. Of more interest are *contraction*, in which an agent reduces its set of beliefs, and *revision*, in which an agent consistently incorporates a new belief. These operators can be characterised by two means. First, a set of *rationality postulates* for a belief change function may be provided; these postulates stipulate constraints that should govern any rational belief change function. Second, specific constructions for a belief change function are given. *Representation results* can then be given (or at least are highly desirable) showing that a set of rationality postulates exactly captures the operator given by a particular construction.

Our focus in this paper is on belief contraction, and so we review these notions with respect to this operator. Informally, the contraction of a belief set by a formula is a belief set in which that formula is not believed. Formally, a contraction function $\dot{-}$ is a function from $2^{\mathcal{L}} \times \mathcal{L}$ to $2^{\mathcal{L}}$ satisfying the following postulates:

($K\dot{-}1$) $K\dot{-}\phi$ is a belief set.

($K\dot{-}2$) $K\dot{-}\phi \subseteq K$.

($K\dot{-}3$) If $\phi \notin K$, then $K\dot{-}\phi = K$.

($K\dot{-}4$) If not $\vdash \phi$, then $\phi \notin K\dot{-}\phi$.

($K\dot{-}5$) If $\phi \in K$, then $K \subseteq (K\dot{-}\phi) + \phi$.

($K\dot{-}6$) If $\vdash \phi \equiv \psi$, then $K\dot{-}\phi = K\dot{-}\psi$.

($K\dot{-}7$) $K\dot{-}\phi \cap K\dot{-}\psi \subseteq K\dot{-}(\phi \wedge \psi)$.

($K\dot{-}8$) If $\psi \notin K\dot{-}(\psi \wedge \phi)$ then $K\dot{-}(\phi \wedge \psi) \subseteq K\dot{-}\psi$.

The first six postulates are called the *basic* contraction postulates, while the last two are referred to as the *supplementary* postulates. We have the following informal interpretations of the postulates: contraction yields a belief set ($K\dot{-}1$) in which the sentence for contraction ϕ is not believed (unless ϕ is a tautology) ($K\dot{-}4$). No new sentences are believed ($K\dot{-}2$), and if the formula is not originally believed then contraction has no effect ($K\dot{-}3$). The fifth postulate, the so-called *recovery* postulate, states that nothing is lost if one contracts and expands by the same sentence. This postulate is controversial; see for example [Hansson, 1999]. The sixth postulate asserts that contraction is independent of how a sentence is expressed. The last two postulates express relations between contracting by conjunctions and contracting by the constituent conjuncts. ($K\dot{-}7$) says that if a formula is in the result of contracting by each of two formulas then it is in the result of contracting by their conjunction. ($K\dot{-}8$) says that if a conjunct is not in the result of contracting by a conjunction, then contracting by that conjunct is (using ($K\dot{-}7$)) the same as contracting by the conjunction.

Several constructions have been proposed to characterise belief change. The original construction was in terms of *remainder sets*, where a remainder set of K with respect to ϕ is a maximal subset of K that fails to imply ϕ. Formally:

Definition 1 *Let $K \subseteq \mathcal{L}$ and let $\phi \in \mathcal{L}$.*
$K \downarrow \phi$ is the set of sets of formulas s.t. $K' \in K \downarrow \phi$ iff

1. $K' \subseteq K$
2. $K' \not\vdash \phi$
3. *For any K'' s.t. $K' \subset K'' \subseteq K$, it holds that $K'' \vdash \phi$.*

$X \in K \downarrow \phi$ is a remainder set of K wrt ϕ.

From a logical point of view, the remainder sets comprise equally-good candidates for a contraction function. *Selection functions* are introduced to reflect the extra-logical factors that need to be taken into account, to obtain the "best" or most plausible remainder sets. In *maxichoice contraction*, the selection function determines a single selected remainder set as the contraction. In partial meet contraction, the selection function returns a subset of the remainder sets, the intersection of which constitutes the contraction. Thus if the selection function is denoted by $\gamma(\cdot)$, then the contraction of K by formula ϕ can be expressed by

$$K\dot{-}\phi \;=\; \bigcap \gamma(K \downarrow \phi).$$

For arbitrary theory K and function $\dot{-}$ from $2^{\mathcal{L}} \times \mathcal{L}$ to $2^{\mathcal{L}}$, it proves to be the case that $\dot{-}$ is a partial meet contraction function iff it satisfies the basic contraction postulates ($K\dot{-}1$)–($K\dot{-}6$). Last, let $\preceq$ be a transitive relation on 2^K, and let the selection function be defined by:

$$\gamma(K \downarrow \phi) \;=\; \{K' \in K \downarrow \phi \mid \forall K'' \in K \downarrow \phi, K'' \preceq K'\}.$$

γ is a *transitively relational* selection function, and $\dot{-}$ defined in terms of such a γ is a *transitively relational partial meet contraction function*. Then we have:

Theorem 1 ([Alchourrón *et al.*, 1985]) *Let K be a belief set and let $\dot{-}$ be a function from $2^{\mathcal{L}} \times \mathcal{L}$ to $2^{\mathcal{L}}$. Then*

1. *$\dot{-}$ is a partial meet contraction function iff it satisfies the contraction postulates ($K\dot{-}1$)–($K\dot{-}6$).*
2. *$\dot{-}$ is a transitively relational partial meet contraction function iff it satisfies the contraction postulates ($K\dot{-}1$)–($K\dot{-}8$).*

The second major construction for contraction functions is called *epistemic entrenchment*. The general idea is that extra-logic factors related to contraction are given by an ordering on formulas in the agent's belief set, reflecting how willing the agent would be to give up a formula. Then a contraction function can be defined in terms of removing less entrenched formulas from the belief set. It is shown in [Gärdenfors and Makinson, 1988] that for logics including classical propositional logic, the two types of constructions, selection functions over remainder sets and epistemic entrenchment orderings, capture the same class of contraction functions; see also [Gärdenfors, 1988] for details.

3 Horn Theories and Horn Contraction

3.1 Preliminary Considerations

Let $\mathbf{P} = \{a, b, c, \dots\}$ be a finite set of atoms, or propositional letters, that includes the distinguished atom $\bot$. $\mathcal{L}_H$ is the language of *Horn formulas*. That is, $\mathcal{L}_H$ is given by:

1. Every $p \in \mathbf{P}$ is a Horn clause.
2. $a_1 \wedge a_2 \wedge \cdots \wedge a_n \rightarrow a$, where $n \geq 0$, and a, a_i ($1 \leq i \leq n$) are atoms, is a Horn clause.
3. Every Horn clause is a Horn formula.
4. If ϕ and ψ are Horn formulas then so is $\phi \wedge \psi$.

For a rule r as in 2 above, $head(r)$ is a, and $body(r)$ is the set $\{a_1, a_2, \dots, a_n\}$. Allowing conjunctions of rules, as given in 4, adds nothing of interest to the expressivity of the language with respect to reasoning. However, it adds to the expressibility of contraction, as we are able to contract by more than a single Horn clause. For convenience, we use $\top$ to stand for some arbitrary tautology.

An *interpretation* of $\mathcal{L}_H$ is a function from $\mathbf{P}$ to $\{true, false\}$ such that $\bot$ is assigned $false$. Sentences of $\mathcal{L}_H$ are *true* or *false* in an interpretation according to the standard rules in propositional logic. An interpretation M is a *model* of a sentence ϕ (or set of sentences), written $M \models \phi$, just if M makes ϕ true. $Mod(\phi)$ is the set of models of formula (or set of formulas) ϕ; thus $Mod(\top)$ is the set of interpretations of $\mathcal{L}_H$. An interpretation is usually identified with the atoms true in that interpretation. Thus, for $\mathbf{P} = \{p, q, r, s\}$ the interpretation $\{p, q\}$ is that in which p and q are true and r and s are false. For convenience, we also express interpretations by juxtaposition of atoms. Thus the interpretations $\{\{p, q\}, \{p\}, \{\}\}$ will usually be written as $\{pq, p, \emptyset\}$.

A key point is that Horn theories are characterised semantically by the fact that the models of a Horn theory are closed under intersections of positive atoms in an interpretation. That is, Horn theories satisfy the constraint:

$$\text{If } M_1, M_2 \in Mod(H) \text{ then } M_1 \cap M_2 \in Mod(H).$$

This leads to the notion of the *characteristic models* [Khardon, 1995] of a Horn theory: M is a characteristic model of theory H just if for every $M_1, M_2 \in Mod(H)$, $M_1 \cap M_2 = M$ implies that $M = M_1$ or $M = M_2$. E.g. the theory expressed by $\{p \wedge q \rightarrow \bot, r\})$ has models $\{pr, qr, r\}$ and characteristic models $\{pr, qr\}$. Since $pr \cap qr = r$, r isn't a characteristic model of H.

A Horn formula ψ is entailed by a set of Horn formulas A, $A \vdash_H \psi$, just if any model of A is also a model of ψ. For simplicity, and because we work exclusively with Horn formulas, we drop the subscript and write $A \vdash \psi$. If $A = \{\phi\}$ is a singleton set then we just write $\phi \vdash \psi$. A set of formulas A is *inconsistent* just if $A \vdash \bot$. We use $\phi \leftrightarrow \psi$ to represent logical equivalence, that is $\phi \vdash \psi$ and $\psi \vdash \phi$.

Notation: We collect here notation that is used in the paper. Lower-case Greek characters ϕ, ψ, ..., possibly subscripted, denote arbitrary formulas of $\mathcal{L}_H$. Upper case Roman characters A, B, ..., possibly subscripted, denote arbitrary sets of formulas. H (H_1, H', etc.) denotes Horn belief sets, so that $\phi \in H$ iff $H \vdash_H \phi$.

$Cn^h(A)$ is the deductive closure of a Horn formula or set of formulas A under Horn derivability. $|\phi|$ is the set of maximal, consistent Horn theories that contain ϕ. m (and subscripted variants) represents a maximum consistent set of Horn formulas.

M (M_1, M', etc.) denote interpretations over some fixed language. $Mod(A)$ is the set of models of A. Arbitrary sets of interpretations will be denoted $\mathcal{M}$ ($\mathcal{M}'$ etc.). $Cl_\cap(\mathcal{M})$ is the intersection closure of a set of interpretations $\mathcal{M}$;[1] that is, $Cl_\cap(\mathcal{M})$ is the least set such that $\mathcal{M} \subseteq Cl_\cap(\mathcal{M})$ and M_1, $M_2 \in Cl_\cap(\mathcal{M})$ implies that $M_1 \cap M_2 \in Cl_\cap(\mathcal{M})$. Note that M denotes an interpretation expressed as a set of atoms, while m denotes a maximum consistent set of Horn formulas. Thus the logical content is the same, in that an interpretation defines a maximum consistent set of Horn formulas, and vice versa. We retain these two interdefinable notations, since each is useful in the subsequent development. Similar comments apply to $Mod(\phi)$ vs. $|\phi|$.

Since **P** is finite, a (Horn or propositional logic) belief set may be finitely represented, that is, for X a belief set, there is a formula ϕ such that $Cn^h(\phi) = X$. As well, we make use of the fact that there is a 1-1 correspondence between elements of $|\phi|$ and of $Mod(\phi)$.

[1]Recall that an interpretation is represented by the set of atoms true in the interpretation.

counter-model	induced models	resulting KB	r.s.
a		$a \wedge (c \rightarrow b)$	$\surd$
ac	a	a	
b		$b \wedge (c \rightarrow a)$	$\surd$
bc	b	b	
$\emptyset$		$(a \rightarrow b) \wedge (b \rightarrow a) \wedge (c \rightarrow a \wedge b)$	$\surd$
c	$\emptyset$	$(a \rightarrow b) \wedge (b \rightarrow a)$	

Figure 1: Example: Candidates for Horn contraction

3.2 Horn Contraction

The last few years have seen work on Horn contraction. Delgrande [2008] addressed *maxichoice Horn belief set contraction* based on (Horn) remainder sets, called *e-remainder sets*. The definition of *e-remainder sets* for Horn clause belief sets is the same as that for a remainder set (Definition 1) but with respect to Horn clauses and Horn derivability. For H a Horn belief set and $\phi \in \mathcal{L}_H$, the set of *e-remainder sets* with respect to H and ϕ is denoted by $H \downarrow_e \phi$.

Booth, Meyer, and Varzinczak [2009] subsequently investigated this area by considering partial meet contraction, as well as a generalisation of partial-meet, based on the idea of *infra-remainder sets* and package contraction. In [Booth *et al.*, 2009], an *infra remainder sets* is defined as follows:

Definition 2 *For belief sets H and X, $X \in H \Downarrow_e \phi$ iff there is some $X' \in H \downarrow_e \phi$ such that $(\bigcap H \downarrow_e \phi) \subseteq X \subseteq X'$. The elements of $H \Downarrow_e \phi$ are the* infra e-remainder sets *of H with respect to ϕ.*

All e-remainder sets are infra e-remainder sets, as is the intersection of any set of e-remainder sets. It proved to be the case that e-remainder sets (and including the infra-remainder sets of [Booth *et al.*, 2009]) are not sufficiently expressive for contraction.

The problem arises from the relation between remainder sets on the one hand, and their counterpart in terms of interpretations on the other. In the classical AGM approach, a remainder set is characterised semantically by a minimal superset of the models of the agent's belief set such that this superset does not entail the formula for contraction. As a result, the models of a remainder set consist of the models of a belief set H together with a countermodel of the formula ϕ for contraction. With Horn clauses, things are not quite so simple, in that for a countermodel M of ϕ, there may be no Horn remainder set that has M as a model.

To see this, consider the following example, adapted from [Delgrande and Wassermann, 2010].

Example 1 *Let $\mathbf{P} = \{a, b, c\}$ and $H = Cn^h(a \wedge b)$. Consider candidates for $H \dot{-} (a \wedge b)$. There are three remainder sets, given by the Horn closures of $a \wedge (c \rightarrow b)$, $b \wedge (c \rightarrow a)$, and $(a \rightarrow b) \wedge (b \rightarrow a) \wedge (c \rightarrow a \wedge b))$. Any infra-remainder set contains the closure of $(c \rightarrow a) \wedge (c \rightarrow b)$.*

See Figure 1. In the first line of the table, we have that a (viz. $\{a, \neg b, \neg c\}$) is a countermodel of $a \wedge b$. Adding this model to the models of H yields the models of the formula $a \wedge (c \rightarrow b)$. This characterises a remainder set, as indicated in the last column. In the second line, we have that ac (viz.

$\{a, \neg b, c\}$) is another countermodel of H. However, since H has a model ab, the intersection of these models, $ab \cap ac = a$ must also be included; this is the item in the second column. The resulting belief set is characterised by the interpretations $Mod(H) \cup \{ac, a\} = \{abc, ab, ac, a\}$, which is the set of models of formula a, as given in the third column. However, the result isn't a remainder set, since $Cn^h(a \wedge (c \rightarrow b))$ is a logically stronger belief set than $Cn^h(a)$, which also fails to imply $a \wedge b$.

This result is problematic for both [Delgrande, 2008] and [Booth *et al.*, 2009]. For example, in none of the approaches in these papers is it possible to obtain $H \dot{-}_e (a \wedge b) \leftrightarrow a$, nor $H \dot{-}_e (a \wedge b) \leftrightarrow (a \equiv b)$. But presumably these possibilities are desirable as *potential* contractions. Thus, in all of the approaches developed in the cited papers, it is not possible to have a contraction wherein $a \wedge \neg b \wedge c$ corresponds to a model of the contraction.

This issue was addressed in [Delgrande and Wassermann, 2010]. There the *characteristic models* of maxichoice candidates for $H \dot{-}_e \phi$ consist of the *characteristic models* of H together with a single interpretation from $Mod(\top) \setminus Mod(\phi)$. The resulting theories, called *weak remainder sets*, corresponded to the theories given in the third column in Figure 1.

Definition 3 ([Delgrande and Wassermann, 2010]) *Let H be a Horn belief set, and let ϕ be a Horn formula.*

$H \Downarrow_e \phi$ is the set of sets of formulas s.t. $H' \in H \Downarrow_e \phi$ iff $H' = H \cap m$ for some $m \in |\top| \setminus |\phi|$.

$H' \in H \Downarrow_e \phi$ is a weak remainder set *of H and ϕ.*

The following characterizations were given for maxichoice and partial meet Horn contraction:

Theorem 2 ([Delgrande and Wassermann, 2010]) *Let H be a Horn belief set. Then $\dot{-}_w$ is an operator of maxichoice Horn contraction based on weak remainders iff $\dot{-}_w$ satisfies the following postulates.*

($H \dot{-}_w 1$) *$H \dot{-}_w \phi$ is a belief set.* *(closure)*

($H \dot{-}_w 2$) *If not $\vdash \phi$, then $\phi \notin H \dot{-}_w \phi$.* *(success)*

($H \dot{-}_w 3$) *$H \dot{-}_w \phi \subseteq H$.* *(inclusion)*

($H \dot{-}_w 4$) *If $\phi \notin H$, then $H \dot{-}_w \phi = H$.* *(vacuity)*

($H \dot{-}_w 5$) *If $\vdash \phi$ then $H \dot{-}_w \phi = H$* *(failure)*

($H \dot{-}_w 6$) *If $\phi \leftrightarrow \psi$, then $H \dot{-}_w \phi = H \dot{-}_w \psi$.* *(extensionality)*

($H \dot{-}_w 7$) *If $H \neq H \dot{-}_w \phi$ then $\exists \beta \in \mathcal{L}_H$ s.t. $\{\phi, \beta\}$ is inconsistent, $H \dot{-}_w \phi \subseteq Cn^h(\{\beta\})$ and $\forall H'$ s.t $H \dot{-}_w \phi \subset H' \subseteq H$ we have $H' \not\subseteq Cn^h(\{\beta\})$.* *(maximality)*

Theorem 3 ([Delgrande and Wassermann, 2010]) *Let H be a Horn belief set. Then $\dot{-}_w$ is an operator of partial meet Horn contraction based on weak remainders iff $\dot{-}_w$ satisfies the postulates ($H \dot{-}_w 1$) – ($H \dot{-}_w 6$) and:*

($H \dot{-}_{pm} 7$) *If $\beta \in H \setminus (H - \alpha)$, then there is some H' such that $H - \alpha \subseteq H'$, $\alpha \notin Cn^h(H')$ and $\alpha \in Cn^h(H' \cup \{\beta\})$* *(weak relevance)*

More recently, [Zhuang and Pagnucco, 2010b] have addressed Horn contraction from the point of view of epistemic entrenchment. They compare AGM contraction via epistemic entrenchment in classical propositional logic with contraction in Horn logics. A postulate set is provided and shown to characterise entrenchment-based Horn contraction. The fact that AGM contraction refers to disjunctions of formulas, which in general will not be Horn, is handled by considering *Horn strengthenings* in their postulate set, which is to say, logically weakest Horn formulas that subsume the disjunction. In contrast to earlier work, their postulate set includes equivalents to the supplemental postulates, and so goes beyond the set of basic postulates.

For a given clause φ, the set of its Horn strengthenings $(\varphi)_H$ is the set such that $\psi \in (\varphi)_H$ if and only if ψ is a Horn clause and there is no Horn clause ψ' such that $\psi \subset \psi' \subseteq \varphi$.

Of the set of ten postulates given in [Zhuang and Pagnucco, 2010b], five correspond to postulates characterizing partial meet contraction based on weak remainders as defined in [Delgrande and Wassermann, 2010] and two correspond to the supplementary postulates ($K \dot{-} 7$) and ($K \dot{-} 8$). The three new postulates are:

($H \dot{-} 5$) If $\psi \in H \dot{-} \varphi \wedge \psi$ then $\psi \in H \dot{-} \varphi \wedge \psi \wedge \delta$

($H \dot{-} 9$) If $\psi \in H \setminus H \dot{-} \varphi$ then $\forall \chi \in (\varphi \vee \psi)_H, \chi \notin H \dot{-} \varphi$

($H \dot{-} 10$) If $\forall \chi \in (\varphi \vee \psi)_H, \chi \notin H \dot{-} \varphi \wedge \psi$ then $\psi \notin H \setminus H \dot{-} \varphi$

While there has been other work on belief change and Horn logic, such work focussed on specific aspects of the problem, rather than a general characterisation of Horn clause belief change. For example, Eiter and Gottlob [1992] address the complexity of specific approaches to revising knowledge bases, including the case where the knowledge base and formula for revision are conjunctions of Horn clauses. Not unexpectedly, results are generally better in the Horn case. Liberatore [2000] considers the problem of compact representation for revision in the Horn case. Basically, given a knowledge base K and formula ϕ, both Horn, the main problem addressed is whether the knowledge base, revised according to a given operator, can be expressed by a propositional formula whose size is polynomial with respect to the sizes of K and ϕ. [Langlois *et al.*, 2008] approaches the study of revising Horn formulas by characterising the existence of a complement of a Horn consequence; such a complement corresponds to the result of a contraction operator. This work may be seen as a specific instance of a general framework developed in [Flouris *et al.*, 2004]. In [Flouris *et al.*, 2004], belief change is studied under a broad notion of *logic*, where a logic is a set closed under a Tarskian consequence operator. In particular, they give a criterion for the existence of a contraction operator satisfying the basic AGM postulates in terms of *decomposability*.

4 Supplementary postulates

In this section we investigate how the different proposals for Horn contraction operations behave with respect to the supplementary postulates (K-7) and (K-8). Throughout the section, we consider all selection functions to be transitively relational.

First we consider the operation of Horn Partial Meet e-Contraction as defined in [Delgrande, 2008]. The following example shows that, considering $\downarrow_e$ as defined in [Delgrande, 2008], Horn Partial Meet e-Contraction does not satisfy $(K\dot{-}7)$:

Example 2 *Let* $H = Cn^h(\{a \to b, b \to c, a \to d, d \to c\})$.
We then have

$H \downarrow_e a \to c = \{H_1, H_2, H_3, H_4\}$
$H \downarrow_e b \to c = \{H_5\}$

where:
$H_1 = Cn^h(\{a \to b, a \to d\})$,
$H_2 = Cn^h(\{a \to b, a \wedge c \to d, d \to c\})$,
$H_3 = Cn^h(\{b \to c, a \wedge c \to b, a \to d\})$,
$H_4 = Cn^h(\{a \wedge c \to b, b \to c, a \wedge c \to d, d \to c, a \wedge d \to b, a \wedge b \to d\})$, *and*
$H_5 = Cn^h(\{a \to b, a \to d, d \to c\})$

Note that the two first elements of $H \downarrow_e a \to c$ *are subsets of the single element of* $H \downarrow_e b \to c$ *and hence, cannot belong to* $H \downarrow_e a \to c \wedge b \to c$.

$H \downarrow_e a \to c \wedge b \to c = \{H_3, H_4, H_5\}$

If we take a selection function based on a transitive relation between remainder sets that gives priority in the order in which they appear in this example, i.e., $H_5 \prec H_4 \prec H_3 \prec H_2 \prec H_1$*, we will have:*

$H - a \to c = H_1$
$H - b \to c = H_5$
$H - a \to c \wedge b \to c = H_3$

And we see that $H - a \to c \cap H - b \to c = H_1 \not\subseteq H_3 = H - a \to c \wedge b \to c$

The same example shows that the operation does not satisfy $(K\dot{-}8)$:

$a \to c \notin H - a \to c \wedge b \to c$, but $H - a \to c \wedge b \to c \not\subseteq H - a \to c$.

If there are no further restrictions on the selection function, the same example also shows that contraction based on infra-remainders does not satisfy the supplementary postulates. Note that each remainder set in the example is also an infra-remainder and that the selection function always selects a single element. It suffices to assign all the remaining infra-remainders lower priority.

Now we can show that the operation of partial meet based on weak remainders (PMWR) has a better behaviour with respect to the supplementary postulates:

Proposition 1 *Partial meet based on weak remainders and a transitive relational selection function satisfies* $(K\dot{-}7)$ *and* $(K\dot{-}8)$.

We have seen that Epistemic Entrenchment Horn Contraction (EEHC) is characterized by a set of ten postulates. In [Zhuang and Pagnucco, 2010a], it is shown that transitively relational PMWR as defined above is more general than EEHC. This means that any operation satisfying their set of 10 postulates (which include $(K\dot{-}7)$ and $(K\dot{-}8)$) is a PMWR. We have seen that PMWR satisfies $(K\dot{-}7)$ and $(K\dot{-}8)$, hence, in order to compare PMWR and EEHC, we need to know whether PMWR satisfies $(H\dot{-}5)$, $(H\dot{-}9)$ and $(H\dot{-}10)$.

Proposition 2 *PMWR satisfies* $(H\dot{-}5)$.

Proposition 3 *PMWR satisfies* $(H\dot{-}9)$

PMWR in general does not satisfy $(H\dot{-}10)$, as the following example shows.

Let $H = Cn^h(\{a, b\})$. Then
$H \Downarrow_e a = \{H_1, H_3\}$ and
$H \Downarrow_e a \wedge b = \{H_1, H_2, H_3\}$, where
$H_1 = Cn^h(\{a \vee \neg b, b \vee \neg a\})$,
$H_2 = Cn^h(\{a\})$ and
$H_3 = Cn^h(\{b\})$.

Assuming a selection function based on a transitive relation such that $H_1 \prec H_2$ and $H_1 \prec H_3$ (and $H_2 \preceq H_3$ and $H_3 \preceq H_2$), we have

$H - a = H_3$ and $H - a \wedge b = H_2 \cap H_3$

Since $(a \vee b)_H = \{a, b\}$, we have that for any $\chi \in (a \vee b)_H$, $\chi \notin H - a \wedge b$, but $b \in H - a$.

In order to finish the comparison between the sets of postulates, it is interesting to note the following:

Observation 1 $(H\dot{-}9)$ *implies weak relevance.*

5 Package Contraction

In this section we consider *Horn package contraction*. For belief set H and a set of formulas Φ, the package contraction $H \dot{-}_p \Phi$ is a form of contraction in which no member of Φ is in $H \dot{-}_p \Phi$. As [Booth *et al.*, 2009] points out, this operation is of interest in Horn clause theories given their limited expressivity: in order to contract by ϕ and ψ simultaneously, one cannot contract by the disjunction $\phi \vee \psi$, since the disjunction is generally not a Horn clause. Hence, one expresses the contraction of both ϕ and ψ as the package contraction $H \dot{-}_p \{\phi, \psi\}$.

We define the notion of Horn package contraction, and show that it is in fact expressible in terms of maxichoice Horn contraction.

Definition 4 *Let* H *be a Horn belief set, and let* $\Phi = \{\phi_1, \ldots, \phi_n\}$ *be a set of Horn formulas.*
$H \Downarrow_p \Phi$ *is the set of sets of formulas s.t.* $H' \in H \Downarrow_p \Phi$ *iff* $\exists m_1, \ldots, m_n$ *such that, for* $1 \leq i \leq n$*:*
$m_i \in |\top| \setminus |\phi_i|$ *if* $\not\vdash \phi_i$, *otherwise* $m_i = \mathcal{L}_H$
and $H' = H \cap \bigcap_{i=1}^n m_i$.

Definition 5 *Let* γ *be a selection function on* H *such that* $\gamma(H \Downarrow_p \Phi) = \{H'\}$ *for some* $H' \in H \Downarrow_p \Phi$.
The (maxichoice) package Horn contraction *based on weak remainders is given by:*

$$H \dot{-}_p \Phi = \gamma(H \Downarrow_p \Phi)$$

if $\emptyset \neq \Phi \cap H \not\subseteq Cn^h(\top)$*; and* H *otherwise.*

The following result relates elements of $H \Downarrow_p \Phi$ to weak remainders.

Proposition 4 *Let* H *be a Horn belief set and let* $\Phi = \{\phi_1, \ldots, \phi_n\}$ *be a set of Horn formulas where for* $1 \leq i \leq n$ *we have* $\not\vdash \phi_i$.
Then $H' \in H \Downarrow_p \Phi$ *iff for* $1 \leq i \leq n$ *there are* $H_i \in H \Downarrow_e \phi_i$ *and* $H' = \bigcap_{i=1}^n H_i$.

It follows immediately from this that any maxichoice Horn contraction defines a package contraction, and vice versa.

Example 3 *Consider the Horn belief set* $H = Cn^h(\{a, b\})$ *over* $\mathbf{P} = \{a, b, c\}$. *We want to determine elements of*

$$H \Downarrow_p \Phi \;=\; Cn^h(\{a, b\}) \Downarrow_p \{a, b\}.$$

It proves to be the case that there are a total of 14 elements in $H \Downarrow_p \Phi$ *and so 14 candidate package contractions. We have the following.*

1. *There are 4 countermodels of* a, *given by:*

$$A = \{bc, b, c, \emptyset\}.$$

Thus there are four weak remainders corresponding to these countermodels, and so four candidates for maxichoice Horn contraction by a.

2. *Similarly there are 4 countermodels of* b:

$$B = \{ac, a, c, \emptyset\}.$$

3. *Members of* $H \Downarrow_p \Phi$ *are given by*

$$Cl_\cap(Mod(H) \cup \{x\} \cup \{y\})$$

for $x \in A$ *and* $y \in B$.

For example, for $x = bc$, $y = \emptyset$, *we have that* $Cl_\cap(Mod(H) \cup \{x\} \cup \{y\}) = \{abc, ab, bc, b, \emptyset\}$, *which is the set of models of* $(c \to b) \wedge (a \to b)$.

For $x = bc$, $y = ac$, *we have that* $Cl_\cap(Mod(H) \cup \{x\} \cup \{y\}) = Cn^h(\top)$; *this holds for no other choice of* x *and* y.

What this example indicates informally is that there is a great deal of scope with respect to candidates for package contraction. To some extent, such a combinatorial explosion of possibilities is to be expected, given the fact that a formula will in general have a large number of countermodels, and that this is compounded by the fact that each formula in a package contraction does not hold in the result. However, it can also be noted that some candidate package contractions appear to be excessively weak; for example it would be quite drastic to have $Cn^h(\top)$ as the result of such a contraction. As well, some candidate package contractions appear to contain redundancies, in that a selected countermodel of a may also be a countermodel of b, in which case there seems to be no reason to allow the possible incorporation of a separate countermodel of b. Consequently, we also consider versions of package contraction that in some sense yield a maximal belief set. However, first we provide results regarding package contraction.

We have the following result:

Theorem 4 *Let* H *be a Horn belief set. Then if* $\dot{-}_p$ *is an operator of maxichoice Horn package contraction based on weak remainders then* $\dot{-}_p$ *satisfies the following postulates.*

($H \dot{-}_p 1$) $H \dot{-}_p \Phi$ *is a belief set.* *(closure)*

($H \dot{-}_p 2$) *For* $\phi \in \Phi$, *if not* $\vdash \phi$, *then* $\phi \notin H \dot{-}_p \Phi$ *(success)*

($H \dot{-}_p 3$) $H \dot{-}_p \Phi \subseteq H$ *(inclusion)*

($H \dot{-}_p 4$) $H \dot{-}_p \Phi = H \dot{-}_p (H \cap \Phi)$ *(vacuity)*

($H \dot{-}_p 5$) $H \dot{-}_p \Phi = H \dot{-}_p (\Phi \setminus Cn^h(\top))$ *(failure)*

($H \dot{-}_p 5b$) $H \dot{-}_p \emptyset = H$ *(triviality)*

($H \dot{-}_p 6$) *If* $\phi \leftrightarrow \psi$, *then*
$H \dot{-}_p (\Phi \cup \{\phi\}) = H \dot{-}_p (\Phi \cup \{\psi\})$ *(extensionality)*

($H \dot{-}_p 7$) *If* $H \neq H \dot{-}_p \Phi$ *then for*

$$\Phi' \;=\; (\Phi \setminus Cn^h(\top)) \cap H \;=\; \{\phi_1, \ldots, \phi_n\}$$

there is $\{\beta_1, \ldots, \beta_n\}$ *s.t.* $\{\phi_i, \beta_i\} \vdash \bot$ *and* $H \dot{-}_p \Phi \subseteq Cn^h(\beta_i)$ *for* $1 \le i \le n$;
and $\forall H'$ *s.t* $H \dot{-}_p \Phi \subset H' \subseteq H$, $\exists \beta_i$ *s.t.* $H' \not\subseteq Cn^h(\beta_i)$.
(maximality)

The following result, which shows that package contraction generalises maxichoice contraction, is not surprising, nor is the next result, which shows that a maxichoice contraction defines a package contraction.

Proposition 5 *Let* $\dot{-}_p$ *be an operator of maxichoice Horn package contraction. Then*

$$H \dot{-} \phi = H \dot{-}_p \Phi \quad \textit{for } \Phi = \{\phi\}$$

is an operator of maxichoice Horn contraction based on weak remainders.

Proposition 6 *Let* $\dot{-}$ *be an operator of maxichoice Horn contraction based on weak remainders. Then*

$$H \dot{-}_p \Phi = \bigcap_{\phi \in \Phi} H \dot{-} \phi$$

is an operator of maxichoice Horn package contraction.

As described, a characteristic of maxichoice package contraction is that there are a large number of members of $H \Downarrow_p \Phi$, some of which may be quite weak logically. Of course, a similar point can be made about maxichoice contraction, but in the case of package contraction we can eliminate some candidates via pragmatic concerns. We have that a package contraction $H \dot{-}_p \Phi$ is a belief set $H' \in H \Downarrow_p \Phi$ such that, informally, models of H' contain a countermodel for each $\phi_i \in \Phi$ along with models of H. In general, some interpretations will be countermodels of more than one member of Φ, and so pragmatically, one can select minimal sets of countermodels. Hence in the case that $\bigcap_i (Mod(\top) \setminus Mod(\phi_i)) \neq \emptyset$, a single countermodel, that is some $m \in \bigcap_i (Mod(\top) \setminus Mod(\phi_i))$, would be sufficient to yield a package contraction.

Now, it may be that $\bigcap_i (Mod(\top) \setminus Mod(\phi_i))$ is empty. A simple example illustrates this case:

Example 4 *Let* $H = Cn^h(a \to b, b \to a)$ *where* $\mathbf{P} = \{a, b\}$. *Then* $H \dot{-}_p \{a \to b, b \to a\} = Cn^h(\top)$. *That is, the sole countermodel of* $a \to b$ *is* $\{a\}$ *while that of* $b \to a$ *is* $\{b\}$. *The intersection closure of these interpretations with those of* H *is* $\{ab, a, b, \emptyset\} = Mod(\top)$.

Informally then one can select a minimal set of models such that a countermodel of each member of Φ is in the set. These considerations yield the following definition:

Definition 6 *Let* H *be a Horn belief set, and let* $\Phi = \{\phi_1, \ldots, \phi_n\}$ *be a set of Horn formulas.*

$HS(\Phi)$, *the set of (minimal)* hitting sets *of interpretations with respect to* Φ, *is defined by:*

$S \in HS(\Phi)$ *iff*

1. $S \subseteq |\top|$
2. $S \cap (|\top| \setminus |\phi_i|) \neq \emptyset$ *for* $1 \leq i \leq n$.
3. *For* $S' \subset S$, $S' \cap (|\top| \setminus |\phi_i|) = \emptyset$ *for some* $1 \leq i \leq n$.

Thus we look for sets of sets of interpretations, elements of such a set S are interpretations represented as maximum consistent sets of formulas (Condition 1). As well, this set S contains a countermodel for each member of Φ (2) and moreover S is a subset-minimal set that satisfies these conditions (3). The notion of a hitting set is not new; see [Garey and Johnson, 1979] and see [Reiter, 1987] for an early use in AI. Thus $S \in HS(\Phi)$ corresponds to a minimal set of countermodels of members of Φ.

Definition 7 *$H \Downarrow_p \Phi$ is the set of sets of formulas s.t.*
$H' \in H \Downarrow_p \Phi$ iff $H' = H \cap \bigcap_{m \in S}$ for some $S \in HS(\Phi)$.

Proposition 7 *For $H' \in H \Downarrow_p \Phi$, H' is an operator of maxichoice Horn package contraction.*

Example 5 *Consider where $H = Cn^h(a, b)$, $\mathbf{P} = \{a, b, c\}$.*

1. *Let $\Phi = \{a, b\}$. We obtain that*

$$\begin{aligned} H \Downarrow_p \Phi \quad = \quad & \{\ Cn^h(\top),\ Cn^h(c \rightarrow a),\ Cn^h(c \rightarrow b), \\ & Cn^h(c \rightarrow a, c \rightarrow b), \\ & Cn^h(a \rightarrow b, b \rightarrow a), \\ & Cn^h(a \rightarrow b, b \rightarrow a, c \rightarrow a, c \rightarrow b)\ \}. \end{aligned}$$

Compare this with Example 3, where we have 14 candidate package contractions.

2. *Let $\Phi = \{a, a \wedge b\}$. We obtain that*

$$\begin{aligned} H \Downarrow_p \Phi \quad = \quad & \{\ Cn^h(b),\ Cn^h(b \wedge (c \rightarrow a)), \\ & Cn^h(a \rightarrow b, b \rightarrow a), \\ & Cn^h(a \rightarrow b, b \rightarrow a, c \rightarrow a, c \rightarrow b)\ \}. \end{aligned}$$

Any set of formulas that satisfies Definition 7 clearly also satisfies Definition 5. One can further restrict the set of candidate package contractions by replacing $S' \subset S$ by $|S'| < |S|$ in the third part of Definition 7. As well, of course, one could continue in the obvious fashions to define a notion of partial meet Horn package contraction.

6 Forgetting in Horn Formulas

This section examines another means of removing beliefs from an agent's belief set, that of *forgetting* [Lin and Reiter, 1994; Lang and Marquis, 2002]. Forgetting is an operation on belief sets and atoms of the language; the result of forgetting an atom can be regarded as decreasing the language by that atom.

In general it will be easier to work with a set of Horn clauses, rather than Horn formulas. Since there is no confusion, we will freely switch between sets of Horn clauses and the corresponding Horn formula comprising the conjunction of clauses in the set. Thus any time that a set appears as an element in a formula, it can be understood as standing for the conjunction of members of the set. Thus for sets of clauses S_1 and S_2, $S_1 \vee S_2$ will stand for the formula $(\bigwedge_{\phi \in S_1} \phi) \vee (\bigwedge_{\phi \in S_2} \phi)$. Of course, all such sets will be guaranteed to be finite.

We introduce the following notation for this section, where S is a set of Horn clauses.

- $S[p/t]$ is the result of uniformly substituting $t \in \{\bot, \top\}$ for atom p in S.
- $S_{\downarrow p} = \{\phi \in S \mid \phi \text{ does not mention } p\}$

Assume without loss of generality that for $\phi \in S$, that $head(\phi) \notin body(\phi)$.

The following definition adapts the standard definition for forgetting to Horn clauses.

Definition 8 *For set of Horn clauses S and atom p, define $forget(S, p)$ to be $S[p/\bot] \vee S[p/\top]$.*

This is not immediately useful for us, since a disjunction is generally not Horn. However, the next result shows that this definition nonetheless leads to a Horn-definable forget operator. Recall that for clauses c_1 and c_2, expressed as sets of literals where $p \in c_1$ and $\neg p \in c_2$, that the resolvent of c_1 and c_2 is the clause $(c_1 \setminus \{p\}) \cup (c_2 \setminus \{\neg p\})$. As well, recall that if c_1 and c_2 are Horn, then so is their resolvent.

In the following, $Res(S, p)$ is the set of Horn clauses obtained from S by carrying out all possible resolutions with respect to p.

Definition 9 *Let S be a set of Horn clauses and p an atom. Define*

$$\begin{aligned} Res(S, p) = \ & \{\phi \mid \exists \phi_1, \phi_2 \in S \text{ s.t. } p \in body(\phi_1), \\ & p = head(\phi_2), \text{ and} \\ & \phi = (body(\phi_1) \setminus \{p\} \cup body(\phi_2)) \rightarrow head(\phi_1)\} \end{aligned}$$

Theorem 5 $forget(S, p) \ \leftrightarrow\ S_{\downarrow p} \cup Res(S, p)$.

Corollary 1 *Let S be a set of Horn clauses and p an atom. Then $forget(S, p)$ is equivalent to a set of Horn clauses.*

Corollary 2 *Let S_1 and S_2 be sets of Horn clauses and p an atom. Then $S_1 \leftrightarrow S_2$ implies that $forget(S_1, p) \leftrightarrow forget(S_2, p)$.*

There are several points of interest about these results. The theorem is expressed in terms of arbitrary sets of Horn clauses, and not just deductively-closed Horn belief sets. Hence the second corollary states a principle of irrelevance of syntax for the case for forgetting for belief bases. As well, the expression $S_{\downarrow p} \cup Res(S, p)$ is readily computable, and so the theorem in fact provides a means of computing $forget$. Further, the approach clearly iterates for more than one atom. We obtain the additional result:

Corollary 3

$$forget(forget(S, p), q) \equiv forget(forget(S, q), p).$$

(In fact, this is an easy consequence of the definition of *forget*.) Given this, we can define for set of atoms A, $forget(S, A) = forget(forget(S, a), A \setminus \{a\})$ where $a \in A$. On the other hand, forgetting an atom may result in a quadratic blowup of the knowledge base.

Finally, it might seem that the approach allows for the definition of a revision operator – and a procedure for computing a revision – by using something akin to the Levi Identity. Let $\mathcal{A}(\phi)$ be the set of atoms appearing in (formula or set of formulas) ϕ. Then:

$$FRevise(S, \phi) \stackrel{def}{=} forget(S, \mathcal{A}(S) \cap \mathcal{A}(\phi)) + \phi.$$

In fact, this *does* yield a revision operator, but an operator that in general is far too drastic to be useful. To see this, consider a taxonomic knowledge base which asserts that whales are fish, $whale \rightarrow fish$. Of course, whales are mammals, but in using the above definition to repair the knowledge base, one would first forget *all* knowledge involving whales. Such an example doesn't demonstrate that there are no reasonable revision operators definable via *forget*, but it does show that a naïve approach is problematic.

7 Conclusions

This paper has collected various results concerning Horn belief set contraction. Earlier work has established a general framework for maxichoice and partial meet Horn contraction. The present paper then extends this work in various ways. We examined issues related to supplementary postulates, developed an approach to package contraction, and explored the related notion of forgetting. For future work, it would be interesting to investigate relationships between remainder-based and entrenchment-based Horn contraction, as well as to explore connections to constructions for (Horn) belief revision.

References

[Alchourrón *et al.*, 1985] C.E. Alchourrón, P. Gärdenfors, and D. Makinson. On the logic of theory change: Partial meet functions for contraction and revision. *Journal of Symbolic Logic*, 50(2):510–530, 1985.

[Booth *et al.*, 2009] Richard Booth, Thomas Meyer, and Ivan José Varzinczak. Next steps in propositional Horn contraction. In *Proceedings of the International Joint Conference on Artificial Intelligence*, Pasadena, CA, 2009.

[Delgrande and Wassermann, 2010] James Delgrande and Renata Wassermann. Horn clause contraction functions: Belief set and belief base approaches. In Fangzhen Lin and Uli Sattler, editors, *Proceedings of the Twelfth International Conference on the Principles of Knowledge Representation and Reasoning*, pages 143–152, Toronto, 2010. AAAI Press.

[Delgrande, 2008] J.P. Delgrande. Horn clause belief change: Contraction functions. In Gerhard Brewka and Jérôme Lang, editors, *Proceedings of the Eleventh International Conference on the Principles of Knowledge Representation and Reasoning*, pages 156–165, Sydney, Australia, 2008. AAAI Press.

[Eiter and Gottlob, 1992] T. Eiter and G. Gottlob. On the complexity of propositional knowledge base revision, updates, and counterfactuals. *Artificial Intelligence*, 57(2-3):227–270, 1992.

[Flouris *et al.*, 2004] Giorgos Flouris, Dimitris Plexousakis, and Grigoris Antoniou. Generalizing the AGM postulates: Preliminary results and applications. In *Proceedings of the 10th International Workshop on Non-Monotonic Reasoning (NMR-04)*, pages 171–179, Whistler BC, Canada, June 2004.

[Gärdenfors and Makinson, 1988] P. Gärdenfors and D. Makinson. Revisions of knowledge systems using epistemic entrenchment. In *Proc. Second Theoretical Aspects of Reasoning About Knowledge Conference*, pages 83–95, Monterey, Ca., 1988.

[Gärdenfors, 1988] P. Gärdenfors. *Knowledge in Flux: Modelling the Dynamics of Epistemic States*. The MIT Press, Cambridge, MA, 1988.

[Garey and Johnson, 1979] M.R. Garey and D.S. Johnson. *Computers and Intractability: A Guide to the Theory of NP-Completeness*. W.H. Freeman and Co., New York, 1979.

[Hansson, 1999] S. O. Hansson. *A Textbook of Belief Dynamics*. Applied Logic Series. Kluwer Academic Publishers, 1999.

[Khardon, 1995] Roni Khardon. Translating between Horn representations and their characteristic models. *Journal of Artificial Intelligence Research*, 3:349–372, 1995.

[Lang and Marquis, 2002] J. Lang and P. Marquis. Resolving inconsistencies by variable forgetting. In *Proceedings of the Eighth International Conference on the Principles of Knowledge Representation and Reasoning*, pages 239–250, San Francisco, 2002. Morgan Kaufmann.

[Langlois *et al.*, 2008] M. Langlois, R.H. Sloan, B. Szörényi, and G. Turán. Horn complements: Towards Horn-to-Horn belief revision. In *Proceedings of the AAAI National Conference on Artificial Intelligence*, Chicago, Il, July 2008.

[Liberatore, 2000] Paolo Liberatore. Compilability and compact representations of revision of Horn knowledge bases. *ACM Transactions on Computational Logic*, 1(1):131–161, 2000.

[Lin and Reiter, 1994] F. Lin and R. Reiter. Forget it! In *AAAI Fall Symposium on Relevance*, New Orleans, November 1994.

[Reiter, 1987] R. Reiter. A theory of diagnosis from first principles. *Artificial Intelligence*, 32(1):57–96, 1987.

[Zhuang and Pagnucco, 2010a] Z. Zhuang and Maurice Pagnucco. Two methods for constructing horn contractions. *AI 2010: Advances in Artificial Intelligence*, pages 72–81, 2010.

[Zhuang and Pagnucco, 2010b] Zhi Qiang Zhuang and Maurice Pagnucco. Horn contraction via epistemic entrenchment. In Tomi Janhunen and Ilkka Niemelä, editors, *Logics in Artificial Intelligence - 12th European Conference (JELIA 2010)*, volume 6341 of *Lecture Notes in Artificial Intelligence*, pages 339–351. Springer Verlag, 2010.