



Probabilistic modal logic <sup>1</sup>

**Author(s):**

Juliana Bueno-Soler

Marcelo Finger

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Juliana Bueno-Soler<sup>\*a</sup>

Marcelo Finger<sup>†b</sup>

<sup>a</sup>*Natural and Human Sciences Center  
Federal University of ABC, Santo André, Brazil*

<sup>b</sup>*Institute of Mathematics and Statistics  
University of São Paulo, Brazil*

The relationship between modality and probability has been very alive, as exemplified in [1] where a probabilistic semantic to modal logic fully preserving its quantificational apparatus is developed. His generalized Kripke models for modal logic are coherent with the central motivations of probabilistic semantics, in the sense of interpreting systems of logic without appealing to truth conditions.

On the other side of the question, logic and probability share many qualitative and quantitative aspects, relevant in the analysis of uncertain reasoning. Among the connections between logic and probability that deserve further clarification, one of the main problems concerns the foundations of conditional probabilistic reasoning.

In an attempt to define flexible logical tools suited for applications of modal logic in philosophical context, G. H. von Wright introduced the logic of conditional modalities in [3] (specially pages 89 to 126). Instead of relying on  $\Box p$  ( $p$  is necessary) and  $\Diamond p$  ( $p$  is possible), von Wright proposed the notions of conditional necessity and possibility:  $N(p/q)$  and  $P(p/q)$ , interpreted as “ $p$  is necessary given  $q$ ” and “ $p$  is possible given  $q$ ”.

The famous David Lewis’s “trivialization result” shows that the bar in conditional probability  $P(p/q)$  cannot be expressed as a connective to be evaluated in classical models. However, there are no attempts in the literature concerning the relationship between modal logic and conditional probability in terms of generalized frames. In [2], a general form of generalized frame has been defined, so as to provide incompleteness results for modal logics. The goal of present note is to study a convenient modification of such frames that could be used to overcome Lewis’s objection.

## References

- [1] C. B. Cross. From worlds to probabilities: A probabilistic semantics for modal logic. *Journal of Philosophical Logic* 22(2) (1993), 169–192.
- [2] J. Bueno-Soler. Completeness and incompleteness for anodic modal logics. *Journal of Applied Non-Classical Logics* 19(3) (2009), 291–310.
- [3] G. H. von Wright. *Logical Studies*. Routledge and Kegan Paul. London. 1957.

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\*juliana.bueno@ufabc.edu.br

†mfinger@ime.usp.br