1^a parte: Problemas de otimização Exercício 1: quadrado de lado $\frac{P}{4}$
Exercício 2: $\frac{4R}{3}$
Exercício 3: base: $\frac{1}{\sqrt{2}}$, altura $\sqrt{2}$
Exercício 4: $(\sqrt{2}, \sqrt{2})$
Exercício 5: $t = 0$
Exercício 6: $V_{\max} = \frac{\pi}{4}$
Exercício 7:
(a) o lado do quadrado deve ser L e o lado do triângulo 0
(b) o lado do quadrado deve ser $4\sqrt{5}\frac{L}{4\sqrt{5}+9}$ e o lado do triângulo $L - 4\sqrt{5}\frac{L}{4\sqrt{5}+9} = -9L\left(4\sqrt{5} - 9\right)$

2^a parte: Fórmula de Taylor Exercício 1:(b) considere $f(x) = e^x$ e $x_0 = 0$.

$P_2(x) = f(0) + f'(0)x + \frac{f''(0)}{2}x^2 = 1 + x + \frac{x^2}{2}$, logo $e^{0.04} \approx 1 + (0.04) + \frac{(0.04)^2}{2} \approx 1.\,040\,8$ e $|E(0.04)| < \frac{e^{0.04}}{3!}(0.04)^3 = 1.\,110\,2 \times 10^{-5}$.

Exercício 2: $P_2(x) = f(0) + f'(0)x + \frac{f''(0)}{2}x^2 = 0$
Exercício 3: $P_5(x) = x - \frac{1}{3}x^3 + \frac{1}{5}x^5$

3^a parte: Primitivas e integrais indefinidas Exercício 1:
(a) $\int \frac{x^2+1}{x}dx = \ln x + \frac{1}{2}x^2 + k$
(b) $\int \frac{e^x+e^{-x}}{2}dx = -\frac{1}{2e^{-x}}\left(e^{2(-x)} - 1\right) + k$
(c) $\int \cos^2(3x)dx = \frac{1}{2}x + \frac{1}{12}\sin 6x + k$
(d) $\int (\sin x + \cos x)^2 dx = x - \frac{1}{2}\cos 2x + k$
(e) $\int \cos^4 x dx = \frac{3}{8}x + \frac{1}{4}\sin 2x + \frac{1}{32}\sin 4x + k$
(f) $\int \tan^2 x dx = \tan x - x + k$
(g) $\int \frac{\cos x + \sin x}{\cos x}dx = x - \ln(\cos x) + k$
(h) $\int \sin(3x)\cos(4x)\cos(5x)dx = \frac{1}{24}\cos 6x - \frac{1}{16}\cos 4x - \frac{1}{8}\cos 2x - \frac{1}{48}\cos 12x + k$
(i) $\int xe^{x^2}dx = \frac{1}{2}e^{x^2} + k$
(j) $\int \frac{x}{\sqrt[3]{1+x^2}}dx = \frac{3}{4}\sqrt[3]{(1+x^2)^2} + k$
(k) $\int \cos^3 x \sin^3 x dx = \frac{1}{192}\cos 6x - \frac{3}{64}\cos 2x + k$
(l) $\int \tan^3 x \sin^2 x dx = -\frac{1}{4}\cos(2x) + 2\ln(\cos x) + \frac{\sec^2 x}{2} + k$
(m) $\int \frac{1}{x \ln x}dx = \ln(\ln x) + k$
(n) $\int \frac{e^x}{\sqrt{1-e^{2x}}}dx = \arcsin(e^x) + k$
(o) $\int \frac{1}{x}\cos(\ln x)dx = \sin(\ln x) + k$
(p) $\int \frac{2}{4-9x^2}dx = \frac{1}{6}\ln\left(x + \frac{2}{3}\right) - \frac{1}{6}\ln\left(x - \frac{2}{3}\right) + k$
(r) $\int x \arctan x dx = \frac{1}{2}\arctan x - \frac{1}{2}x + \frac{1}{2}x^2 \arctan x + k$
(s) $\int \arcsin x dx = \sqrt{1-x^2} + x \arcsin x + k$
(t) $\int \ln(x + \sqrt{1+x^2})dx = x\ln\left(x + \sqrt{x^2+1}\right) - \sqrt{x^2+1} + k$
(u) $\int \sin(\ln x)dx = \frac{1}{2}x\sin(\ln x) - \frac{1}{2}x\cos(\ln x) + k$
Exercício 2:
(a) $\int x^2 \ln x dx = \frac{1}{3}x^3 \ln x - \frac{1}{9}x^3 + k$

(b)$\int e^{-x}\cos 2x dx = -\frac{1}{5}e^{-x}\left(\cos 2x - 2\sin 2x\right) + k$
(c)$\int x^3\cos(x^2)dx = \frac{1}{2}\cos x^2 + \frac{1}{2}x^2\sin x^2 + k$
(d)$\int(\ln x)^2 dx = x\left(\ln^2 x - 2\ln x + 2\right) + k$
Exercício 3: $\int e^{-st}dt = -\frac{1}{s}e^{-st} + k$
Exercício 4:
(a)$\int \frac{x^2}{\sqrt{1-x^2}}dx = \frac{1}{2}\arcsin x - \frac{1}{2}x\sqrt{1-x^2} + k$
(b)$\int \sqrt{-x^2+2x+3}dx = \frac{1}{2}(x-1)\sqrt{-x^2+2x+3} - 2\arcsin(\frac{1-x}{2}) + k$
(c)$\int x^2\sqrt{1-x^2}dx = \frac{1}{8}(x\sqrt{1-x^2}(2x^2-1) + \arcsin x) + k$
(d)$\int \sqrt{9-4x^2}dx = \frac{1}{4}(2\sqrt{9-4x^2}x + 9\arcsin(\frac{2x}{3})) + k$
(e)$\int \frac{1}{1+\sqrt{x}}dx = 2\sqrt{x} - 2\ln(1+\sqrt{x}) + k$
(g)$\int \frac{x^2+1}{2x-x^2}dx = \frac{1}{2}\ln x - x - \frac{5}{2}\ln(x-2) + k$
(h)$\int x(\arctan x)^2 dx = \frac{1}{2}(\ln(x^2+1) + (x^2+1)\arctan^2 x - 2x\arctan x) + k$
(i)$\int \sqrt[3]{1+\sqrt{x}}dx = \frac{3}{14}\sqrt[3]{(1+\sqrt{x})^4}(-3+4\sqrt{x}) + k$
(k)$\int \frac{2\tan x}{2+3\cos x}dx = \ln(3\cos x + 2) - \ln(\cos x) + k$
Exercício 6:
(a)$\int \frac{x^3+x+1}{x^2-x}dx = x - \ln x + 3\ln(x-1) + \frac{1}{2}x^2 + k$
(b)$\int \frac{x+1}{x^2+9}dx = \frac{1}{3}\arctan\frac{1}{3}x + \frac{1}{2}\ln\left(x^2+9\right) + k$
(c)$\int \frac{1}{x^2-x-2}dx = \frac{1}{3}\ln(x-2) - \frac{1}{3}\ln(x+1) + k$
(d)$\int \frac{2}{(x+2)(x-1)^2}dx = \frac{2}{9}\left(-\ln(x-1) + \ln(x+2) - \frac{3}{x-1}\right) + k$
(e)$\int \frac{x^2+1}{(x-2)^3}dx = \frac{11-8x}{2(x-2)^2} + \ln(x-2) + k$
(f)$\int \frac{3x^2+5x+4}{x^3+x^2+x-3}dx = \frac{1}{2}\ln\left(x^2+2x+3\right) + 2\ln(x-1) - \frac{1}{4}\sqrt{2}\pi + \frac{1}{2}\sqrt{2}\arctan\sqrt{2}\left(\frac{1}{2}x+\frac{1}{2}\right)$
$+ k$

4^a parte: Equações diferenciais Exercício 1:
(a)$x(t) = \pm\sqrt{t^2+k}$
(b) $y(x) = \tan(\ln kx)$ com $\frac{-\pi}{2} < \ln kx < \frac{\pi}{2}$
(c)$y(x) = \pi + \arctan(x+k)$
(d)$y(x) = \arctan(x+k)$
(e)$u(v) = c\ln|v|$
(f)$y(x) = -2$ ou $y(x) = \frac{2(ke^{4x}-1)}{1+ke^{4x}}$
(g)$y(x) = ke^{-x} + x - 1$
(h)$y(x) = kx + x^2$
(i)$y(x) = ke^{-2x} + \frac{2}{5}(\sin x + 2\cos x)$
Exercício 2:
(a)$y(t) = -\ln(\frac{1}{e} - t)$
(b)$y(t) = 2,\ t \in \mathbb{R}$
Exercício 3: $y(x) = 2x^2$
Exercício 4: $y(x) = 2x^2$
Exercício 5: $y(x) = \sqrt{x^2+5}$