20 OUT 2017 MAT 0134 (1)

2a) $(x_1, y_1) \oplus (x_2, y_2) = (x_1 + x_2, y_1 + y_2)$

$\alpha \otimes (x, y) = (x, \alpha y)$

$V = \mathbb{R}^2 = \{(x, y) \mid x, y \in \mathbb{R}\}$

$v_1, v_2 \in V_1$
$\alpha_1, \alpha_2 \in \mathbb{R}$
5) $\alpha_1 (v_1 + v_2) = \alpha v_1 + \alpha v_2$
6) $(\alpha_1 + \alpha_2) v_1 = \alpha_1 v_1 + \alpha_2 v_2$
7) $(\alpha_1 \alpha_2) v_1 = \alpha_1 (\alpha_2 v_1)$
8) $1 . v_1 = v_1$

1) $\alpha \otimes ((x_1, y_1) \oplus (x_2, y_2)) =$
$= \alpha \otimes (x_1 + x_2, y_1 + y_2) = (x_1 + x_2, \alpha y_1 + \alpha y_2)$
$\parallel$
$\alpha \otimes (x_1, y_1) + \alpha \otimes (x_2, y_2) =$
$= (x_1, \alpha y_1) + (x_2, \alpha y_2) = (x_1 + x_2, \alpha y_1 + \alpha y_2)$

6) $(\alpha_1 + \alpha_2) \otimes (x_1, y_1) = (x_1, (\alpha_1 + \alpha_2) y_1)$
$\alpha_1 \otimes (x_1, y_1) + \alpha_2 \otimes (x_1, y_1) = (x_1, \alpha_1 y_1) + (x_1, \alpha_2 y_1) =$
$= (2x_1, (\alpha_1 + \alpha_2) y_2) \neq (x_1, (\alpha_1 + \alpha_2) y_1)$
(NÃO É ESPAÇO VETORIAL)

b) $\left.\begin{array}{l} (x_1, y_1) \oplus (x_2 + y_2) = (x_1, y_1) \\ (x_2, y_2) \oplus (x_1, y_1) = (x_2, y_2) \end{array}\right\} (x_1, y_1) \neq (x_2, y_2)$

3c) $V = M_2(\mathbb{R})$ e $S = \{A \in M_2(\mathbb{R}) \mid A$ é inversível$\}$
($\det A \neq 0$)

A, B inversíveis, A+B é inversível?
Não. Ex.: $I_2 + (-I_2) = 0$

f) $V = C(\mathbb{R})$ $S = \{f \in C(\mathbb{R}) \mid \int_0^1 f^2(x)\,dx = 0\}$

$f(x)$ é contínua se $\int_0^1 f^2(x)\,dx = 0$, então $f(x) = 0$
$\forall x \in [0,1]$

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4) $B=\{(a,1,0),(1,a,1),(0,1,a)\}$ é base de $\mathbb{R}^3$?

LI: $\alpha(a,1,0)+\beta(1,a,1)+\gamma(0,1,a)=(0,0,0)$

$$\begin{cases} \alpha a + \beta . 1 = 0 \\ \alpha . 1 + a\beta + \gamma = 0 \\ \quad + \beta . 1 + a\gamma = 0 \end{cases}$$

$$\det \begin{bmatrix} a & 1 & 0 \\ 1 & a & 1 \\ 0 & 1 & a \end{bmatrix} \neq 0 \Rightarrow a^3 - 2a \neq 0$$
$$a(a^2-2) \neq 0 \Rightarrow \begin{cases} a \neq 0 \\ a \neq \pm\sqrt{2} \end{cases}$$

5g) $S' = \left\{ A \in M_2(\mathbb{R}) \mid A \text{ comuta com matriz } \begin{bmatrix} 1 & -2 \\ 1 & 3 \end{bmatrix} \right\}$

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} 1 & -2 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} a_{11}+a_{12} & -2a_{11}+3a_{12} \\ a_{21}+a_{22} & -2a_{21}+3a_{22} \end{bmatrix}$$

$$\begin{bmatrix} 1 & -2 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} a_{11}-2a_{21} & a_{12}-2a_{22} \\ a_{11}+3a_{21} & a_{12}+3a_{22} \end{bmatrix}$$

$$\begin{cases} a_{11}+a_{12} = a_{11}-2a_{21} \\ -2a_{11}+3a_{12} = a_{12}-2a_{22} \\ a_{21}+a_{22} = a_{11}+3a_{21} \\ -2a_{21}+3a_{22} = a_{12}+3a_{22} \end{cases} \Leftrightarrow \begin{cases} a_{12}+2a_{21} = 0 \\ -2a_{11}+2a_{12}+2a_{22} = 0 \\ a_{11}+2a_{21}-a_{22} = 0 \\ -a_{12}-2a_{21} = 0 \end{cases} \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} a_{12} = -2a_{21} \\ a_{11} = -2a_{21}+a_{22} \\ a_{11} = a_{12}+a_{22} \end{cases} \Rightarrow A = \begin{bmatrix} -2a_{21}+a_{22} & -2a_{21} \\ a_{21} & a_{22} \end{bmatrix} =$$

$$= a_{21}\begin{bmatrix} -2 & -2 \\ 1 & 0 \end{bmatrix} + a_{22}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$S = \left\{ \begin{bmatrix} -2 & -2 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right\}$$

Os geradores não são proporcionais. Logo LI

5) $S = \{(x,y,z,u,v) \in \mathbb{R}^5 \mid x-u=v\}$

$(x,y,z,x,x) = x(1,0,0,1,1) + y(0,1,0,0,0) +$
$+ z(0,0,1,0,0)$

$$\underbrace{\alpha(1,0,0,1,1) + \beta(0,1,0,0,0) + \gamma(0,0,1,0,0) = (0,0,0,0,0)}$$

$\alpha = 0 \quad \beta = 0 \quad \gamma = 0$

6) $B = \{1, 2-x, x^2+1, 1+x+x^2\}$ $\quad P_3$

$B_0 = \{1, x, x^2, x^3\}$
$B = \{(1,0,0,0), (2,-1,0,0), (1,0,1,0), (1,1,1,0)\}$

$$\det \begin{vmatrix} 1 & 2 & 1 & 1 \\ 0 & -1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{vmatrix} = 0$$, logo não há como escrever P_3

9 a) $\alpha e^{2x} + \beta x e^{2x} + \gamma x^2 e^{2x} + \delta x^3 e^{2x} = 0$

$x = 0 \Rightarrow \alpha + 0 + 0 + 0 = 0 \Rightarrow \alpha = 0$

$x = 1 \Rightarrow \beta e^2 + \gamma e^2 + \delta e^2 = 0$
$x = -1 \Rightarrow -\beta e^2 + \gamma e^2 - \delta e^2 = 0$ $\Big\} \Rightarrow \gamma = 0$

$x = 2 \Rightarrow 2\beta e^4 + 8\delta e^4 = 0 \Rightarrow \beta + 4\delta = 0$

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$$\begin{matrix}(x=1)\\(x=2)\end{matrix}\left\{\begin{array}{l}\beta+\delta=0\\\beta+4\delta=0\end{array}\right\} \Leftrightarrow \left\{\begin{array}{l}\beta=0\\\delta=0\end{array}\right.$$

10c)

$$\alpha\begin{bmatrix}1&0&2\\0&1&-1\end{bmatrix}+\beta\begin{bmatrix}2&1&1\\1&-2&3\end{bmatrix}+\gamma\begin{bmatrix}1&2&-1\\2&4&3\end{bmatrix}=\begin{bmatrix}0&0&0\\0&0&0\end{bmatrix}$$

$$\left\{\begin{array}{l}\alpha+2\beta+\gamma=0\\\beta+2\gamma=0\\2\alpha+\beta-\gamma=0\\\beta+2\gamma=0\\\alpha-2\beta+4\gamma=0\\-\alpha+3\beta+3\gamma=0\end{array}\right. \Leftrightarrow \left\{\begin{array}{l}\beta=-2\gamma\\\alpha=0\\\gamma=0\\\beta=0\end{array}\right.$$

III.2) $\|u\|=1 \qquad \|v\|=1 \qquad \|u-v\|=2$

$\langle u,v\rangle=?$ $\qquad \|u\|=\sqrt{\langle u,u\rangle}$

$$\|u-v\|^2=\langle u-v,u-v\rangle=$$
$$=\langle u,u\rangle-\langle v,u\rangle-\langle u,v\rangle+\langle v,v\rangle=$$
$$=\|u\|^2-2\langle u,v\rangle+\|v\|^2$$
$$\langle u,v\rangle=\frac{1}{2}\left(\|u\|^2+\|v\|^2-\|u-v\|^2\right)=\frac{1}{2}(2-4)=-1$$

3) $\|v\|=\|u\| \Leftrightarrow \langle u+v,u-v\rangle=0$

$\Rightarrow$ $\|v\|=\sqrt{\langle v,v\rangle}=\sqrt{\langle u,u\rangle}=\|u\| \Leftrightarrow \langle v,v\rangle=\langle u,u\rangle$

$$\langle u+v,u-v\rangle=\cancel{\langle u,u\rangle}+\langle v,u\rangle-\langle u,v\rangle-\cancel{\langle v,v\rangle}=$$
$$=\langle v,u\rangle-\langle v,u\rangle=0$$

$(\Leftarrow)$ $\langle u+v,u-v\rangle=0$

$$0=\langle u,u\rangle-\cancel{\langle u,v\rangle}+\cancel{\langle v,u\rangle}-\langle v,v\rangle \Rightarrow$$
$$\langle u,u\rangle-\langle v,v\rangle=0 \Rightarrow \langle u,u\rangle=\langle v,v\rangle\geq 0$$
$$\|u\|=\sqrt{\langle u,u\rangle}=\sqrt{\langle v,v\rangle}=\|v\|$$

III g) $u=(x_1,x_2) \qquad v=(y_1,y_2)$

$\langle u,v\rangle = x_1y_1 + 2x_1y_1 - 2x_2y_1 + 5x_2y_2$

1) $\langle u_1+u_2, v\rangle = \langle u_1, v\rangle + \langle u_2, v\rangle$

2) $\langle \alpha u, v\rangle = \alpha\langle u, v\rangle$

3) $\langle u, v\rangle = \langle v, u\rangle$

$$\langle u,u\rangle = x_1^2 - 2x_1x_2 - 2x_2x_1 + 5x_2^2 =$$
$$= (x_1^2 - 4x_1x_2 + 4x_2^2) + x_2^2 =$$
$$= (x_1 - 2x_2)^2 + x_2^2 \geq 0$$

$$(x_1-2x_2)^2 + x_2^2 = 0 \Rightarrow \begin{cases} x_1 - 2x_2 = 0 \\ x_2 = 0 \end{cases} \Rightarrow \begin{cases} x_1 = 0 \\ x_2 = 0 \end{cases}$$
$$\Rightarrow u = (0,0)$$

Produto usual $u=(1,2)$

$\langle u,u\rangle = 1+4=5 \qquad \|u\| = \sqrt{5}$

Produto do ex 6.

$\langle u,u\rangle = 1-4-4+5\times 4 = 21-8 = 13 \qquad \|u\| = \sqrt{13}$