MAT2219 – Cálculo Diferencial e Integral III
Respostas da lista de Exercícios 1

Prof. Claudio Gorodski

Primeira parte

1.

a. $\frac{8}{3}$, $\frac{2}{3}$.

b. $2(e^2 - 1)$, 1

c. 1, $ln\frac{3}{2}$

d. $(e-1)^2/2$,

2.

a. 2

b. 1/4

c. 1/2

d. 1

3.

a. $\int_2^4 \int_{y/2}^2 dx\, dy + \int_1^2 \int_1^2 dx\, dy$

b. $\int_0^1 \int_y^{\sqrt[3]{y}} dx\, dy$

c. $\int_0^1 \int_{-\frac{1}{2}ln(y)}^{-ln(y)} dx\, dy$

d. $\int_{-1}^1 \int_{-1}^{-|x|} dy\, dx$

4.

a. $\int_{-3}^3 \int_{|x|}^3 dy\, dx$; $\int_0^3 \int_{-y}^y dx\, dy$.

b. $\int_{-1}^0 \int_0^{-x} dy\, dx$, $\int_0^1 \int_{-1}^{-y} dx\, dy$.

c. $\int_0^2 \int_x^{2x} dy\, dx + \int_2^4 \int_x^4 dy\, dx$; $\int_0^4 \int_{y/2}^y dx\, dy$.

d. $\int_0^4 \int_{y/2}^y dx\, dy + \int_4^8 \int_{y/2}^4 dx\, dy = \int_0^4 \int_x^{2x} dy\, dx$

e. $\int_0^1 \int_0^{bx} dy\, dx + \int_1^2 \int_0^{b(2-x)} dy\, dx = \int_0^b \int_{y/b}^{2-y/b} dx\, dy$

$f.$ $\int_0^a \int_{dx/c}^{bx/a} dydx + \int_a^c \int_{dx/c}^{b+(x-a)(d-b)/(c-a)} dydx$, $\int_0^d \int_{ay/b}^{cy/d} dxdy + \int_d^b \int_{ay/b}^{a+(y-b)(c-a)/(d-b)} dxdy$

5.

$a.$ $f(a,b) - f(a,0) - f(0,b) + f(0,0)$

$b.$ $\int_0^b [f(a,y) - f(0,y)]dy$

6.

$a.$ $\int_0^1 \int_0^1 (2x - 3y + 1)dx\, dy = 1/2$

$b.$ $\int_0^1 \int_0^1 (xe^y - ye^x)dx\, dy = 0$

7. $\int_a^b f(x)dx = \int_a^b \int_0^{f(x)} dy\, dx$

8. 50.000π

9. $\frac{1}{6abc}$

10. $\bar{x} = \frac{3}{14} \int_1^3 \int_1^2 x^3 dx\, dy = 45/28$; $\bar{y} = \frac{3}{14} \int_1^3 \int_1^2 yx^2 dx\, dy = 1/2$

11. Área= $A = 2\int_0^{1/\sqrt{2}} \int_y^{\sqrt{1-y^2}} dx\, dy = \frac{\pi}{4}$. As coordenadas do centroíde vem dadas pelas equacoes $\bar{x} = \frac{1}{A}\left(\int_0^{1/\sqrt{2}} \int_y^{\sqrt{1-y^2}} x\, dx\, dy + \int_{-1/\sqrt{2}}^0 \int_{-y}^{\sqrt{1-y^2}} x\, dx\, dy\right)$; $\bar{y} = \frac{1}{A}\left(\int_0^{1/\sqrt{2}} \int_y^{\sqrt{1-y^2}} y\, dx\, dy + \int_{-1/\sqrt{2}}^0 \int_{-y}^{\sqrt{1-y^2}} y\, dx\, dy\right)$.

Segunda parte

1. (a) $\pi^2/4$, (b) 2, (c) 2π.

2. (a) $e - 1/e$

3. 8/3

4. (a) $\int_0^1 \int_{2-y}^{1+\sqrt{1-y^2}} f(x,y)dxdy$; (b) $\int_{-1}^0 \int_{-\sqrt{4y+4}}^{\sqrt{4y+4}} f(x,y)dxdy + \int_0^8 \int_{-\sqrt{4y+4}}^{2-y} f(x,y)dxdy$; (c) $\int_0^1 \int_{e^y}^e f(x,y)dx\, dy$

5. O centro de massa encontra-se à distância de 2/3 dos lados AB e AD.