1-) a) $\log z = \log(r e^{i\theta}) = \log r + i\theta$ $\exp(\log z) = z$

$\left.\begin{array}{l}\cos\theta = -1\\ \text{sen}\,\theta = 0\end{array}\right\} \Rightarrow \theta = (2n+1)\pi$, $n \in \mathbb{Z}$

$\log(-1) = \log 1 + i(2n+1)\pi$

$\log(-1) = i(2n+1)\pi$ $n \in \mathbb{Z}$

b) $\log i = \log 1 + i\left(\frac{\pi}{2} + 2n\pi\right)$

$\left.\begin{array}{l}\cos\theta = 0\\ \text{sen}\,\theta = 1\end{array}\right\} \Rightarrow \theta = \frac{\pi}{2} + 2n\pi$, $n \in \mathbb{Z}$

$\log i = i\left(\frac{\pi}{2} + 2n\pi\right)$ $n \in \mathbb{Z}$

c) $\log(1-i) = \log\sqrt{2} + i\left(\frac{7\pi}{4} + 2n\pi\right) = \frac{1}{2}\log 2 + i\left(\frac{7\pi}{4} + 2n\pi\right)$

$\left.\begin{array}{l}\cos\theta = \frac{1}{\sqrt{2}}\\ \text{sen}\,\theta = \frac{-1}{\sqrt{2}}\end{array}\right\} \theta = \frac{7\pi}{4} + 2n\pi$, $n \in \mathbb{Z}$

$\log(1-i) = \frac{1}{2}\log 2 + i\left(\frac{7\pi}{4} + 2n\pi\right)$ $n \in \mathbb{Z}$

d) $z^w = \exp(w \log z)$

$(1+i)^i = \exp(i\log(1+i)) =$

$= \exp\left[i\left(\log\sqrt{2} + i\left(\frac{\pi}{4} + 2n\pi\right)\right)\right] =$

$= \exp\left[i\,\frac{\log 2}{2}\right]\exp\left[-\left(\frac{\pi}{4} + 2n\pi\right)\right]$

$\left.\begin{array}{l}\cos\theta = \frac{1}{\sqrt{2}}\\ \text{sen}\,\theta = \frac{1}{\sqrt{2}}\end{array}\right\} \theta = \frac{\pi}{4} + 2n\pi\ ;\ n \in \mathbb{Z}$

$(1+i)^i = \exp\left[-\left(\frac{\pi}{4} + 2n\pi\right)\right]\left(\cos\frac{\log 2}{2} + i\,\text{sen}\,\frac{\log 2}{2}\right)$ $n \in \mathbb{Z}$

e) $z = (-1)^{\frac{1}{\pi}}$

$\log z = \log(-1)^{\frac{1}{\pi}}$

$\log z = \frac{1}{\pi}\log(-1)$

do item ⓐ:

$\log z = \frac{1}{\pi}\, i(2n+1)\pi$

$\log z = i(2n+1)$

$z = \exp[i(2n+1)]$ $; n \in \mathbb{Z}$

$z = \cos(2n+1) + i\,\text{sen}(2n+1)$ $; n \in \mathbb{Z}$

2-) $w = \operatorname{arcsen} z$

$\operatorname{sen} w = z = \dfrac{e^{iw} - e^{-iw}}{2i}$

$z = \dfrac{e^{iw}}{2i} - \dfrac{1}{2i e^{iw}}$

$e^{2iw} - 2iz\, e^{iw} - 1 = 0$

$e^{iw} = \dfrac{2iz \pm (-4z^2+4)^{\frac{1}{2}}}{2} = iz + (1-z^2)^{\frac{1}{2}}$

$iw = \log\left(iz + (1-z^2)^{\frac{1}{2}}\right)$

$w = -i \log\left(iz + (1-z^2)^{\frac{1}{2}}\right)$

$\operatorname{arcsen} z = -i \log\left(iz + (1-z^2)^{\frac{1}{2}}\right)$ //

$$(\operatorname{arcsen} z)' = \left(-i \log\left(iz + (1-z^2)^{\frac{1}{2}}\right)\right)' = \frac{-i}{iz + (1-z^2)^{\frac{1}{2}}}\left[i + \frac{(-2z)}{2(1-z^2)^{\frac{1}{2}}}\right] =$$

$$= \frac{-i}{iz + (1-z^2)^{\frac{1}{2}}}\left[\frac{i(1-z^2)^{\frac{1}{2}} - z}{(1-z^2)^{\frac{1}{2}}}\right] = \frac{1}{iz + (1-z^2)^{\frac{1}{2}}} \cdot \frac{(1-z^2)^{\frac{1}{2}} + iz}{(1-z^2)^{\frac{1}{2}}}$$

$$(\operatorname{arcsen} z)' = \frac{1}{(1-z^2)^{\frac{1}{2}}}$$ //

3-) $w = \arctan z$

$\tan w = z = -i\, \dfrac{e^{iw} - e^{-iw}}{e^{iw} + e^{-iw}}$

$z e^{iw} + \dfrac{z}{e^{iw}} = -i e^{iw} + \dfrac{i}{e^{iw}}$

$z e^{2iw} + z = -i e^{2iw} + i$

$(z+i) e^{2iw} = i - z$

$e^{2iw} = \dfrac{i-z}{i+z}$

$2iw = \log \dfrac{i-z}{i+z}$

$w = -\dfrac{i}{2} \log \dfrac{i-z}{i+z} = \dfrac{i}{2} \log \dfrac{i+z}{i-z}$

$\arctan z = \dfrac{i}{2} \log \dfrac{i+z}{i-z}$ //

$$(\arctan z)' = \left(\frac{i}{2} \log \frac{i+z}{i-z}\right)' = \frac{i}{2} \cdot \frac{i-z}{i+z} \cdot \frac{i-z+(i+z)}{(i-z)^2} = \frac{i}{2} \cdot \frac{2i}{-1-z^2}$$

$$(\arctan z)' = \frac{1}{1+z^2}$$ //

4-) a) $\arctan(2i) = \dfrac{i}{2} \log \dfrac{i+2i}{i-2i} = \dfrac{i}{2} \log(-3) = \dfrac{i}{2}\left[\log 3 + i(2n+1)\pi\right]$

$\left.\begin{array}{l} \cos\theta = \dfrac{-3}{3} = -1 \\ \operatorname{sen}\theta = 0 \end{array}\right\}$ $\theta = (2n+1)\pi$, $n \in \mathbb{Z}$

$\arctan(2i) = \dfrac{i}{2}\left[\log 3 + i(2n+1)\pi\right]$; $n \in \mathbb{Z}$ //

4-) b) $w = \text{arc}\tanh z$

$$\tanh w = z = \frac{e^{w} - e^{-w}}{e^{w} + e^{-w}}$$

$$z e^{w} + \frac{z}{e^{w}} = e^{w} - \frac{1}{e^{w}}$$

$$z e^{2w} + z = e^{2w} - 1$$

$$(z-1) e^{2w} = -(z+1)$$

$$e^{2w} = \frac{1+z}{1-z}$$

$$w = \frac{1}{2} \log \frac{1+z}{1-z}$$

$$\text{arc}\tanh z = \frac{1}{2} \log \frac{1+z}{1-z}$$

$$\text{arc}\tanh 0 = \frac{1}{2} \log 1 = \frac{1}{2}\left[\log 1 + i\, 2n\pi\right] = \frac{1}{2}\, i\, 2n\pi$$

$$\boxed{\left.\begin{array}{l}\cos\theta = 1\\ \operatorname{sen}\theta = 0\end{array}\right\}\ \theta = 2n\pi,\ n \in \mathbb{Z}}$$

$$\underline{\text{arc}\tanh 0 = i n\pi} \ ;\ n \in \mathbb{Z}$$

5-) $\text{Re}\, z > 0$ e $\text{arc}\tan t \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$$\log z = \log r + i\theta$$

$$\log(x + iy) = \log\sqrt{x^2+y^2} + i\, \text{arc}\tan\frac{y}{x}$$

$$\log(x + iy) = \frac{1}{2}\log(x^2+y^2) + i\, \text{arc}\tan\frac{y}{x}$$

$$u(x,y) = \frac{1}{2}\log(x^2+y^2) \qquad v(x,y) = \text{arc}\tan\frac{y}{x}$$

$$\left.\begin{array}{l}\dfrac{\partial u}{\partial x} = \dfrac{1}{2}\,\dfrac{2x}{x^2+y^2} = \dfrac{x}{x^2+y^2}\\[2ex] \dfrac{\partial v}{\partial y} = \dfrac{\frac{1}{x}}{1+\left(\frac{y}{x}\right)^2} = \dfrac{x}{x^2+y^2}\end{array}\right\} \Rightarrow \frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}\ ;\ \forall z \neq 0$$

$\dfrac{\partial u}{\partial x}$ e $\dfrac{\partial v}{\partial y}$ são contínuas em todo plano

$$\left.\begin{array}{l}\dfrac{\partial v}{\partial x} = \dfrac{-\frac{y}{x^2}}{1+\left(\frac{y}{x}\right)^2} = \dfrac{-y}{x^2+y^2}\\[2ex] \dfrac{\partial u}{\partial y} = \dfrac{1}{2}\,\dfrac{2y}{x^2+y^2} = \dfrac{y}{x^2+y^2}\end{array}\right\} \Rightarrow \frac{\partial v}{\partial x} = -\frac{\partial u}{\partial y}\ ;\ \forall z \neq 0$$

$\dfrac{\partial v}{\partial x}$ e $\dfrac{\partial u}{\partial y}$ são contínuas em todo plano

$\Rightarrow$ As Condições de Cauchy-Riemann são satisfeitas $\forall z \neq 0$

$\therefore$ $\log z$ é analítica na região $\text{Re}\, z > 0$