MAT0211-45 - Cálculo III
Respostas da Lista de Exercícios 2

1. (a)

$$Jf(r,\theta,\varphi) = \begin{bmatrix} \cos\theta \operatorname{sen}\varphi & -r\operatorname{sen}\theta\operatorname{sen}\varphi & r\cos\theta\cos\varphi \\ \operatorname{sen}\theta\operatorname{sen}\varphi & r\cos\theta\operatorname{sen}\varphi & r\operatorname{sen}\theta\cos\varphi \\ \cos\varphi & 0 & -r\operatorname{sen}\varphi \end{bmatrix}$$

(b) $\det Jf(r,\theta,\varphi) = -r^2 \operatorname{sen}\varphi$.

(c)

$$\begin{aligned} \frac{\partial u}{\partial r}(r,\theta,\varphi) &= \cos\theta\operatorname{sen}\varphi\frac{\partial g}{\partial x} + \operatorname{sen}\theta\operatorname{sen}\varphi\frac{\partial g}{\partial y} + \cos\varphi\frac{\partial g}{\partial z}, \\ \frac{\partial u}{\partial \theta}(r,\theta,\varphi) &= -r\operatorname{sen}\theta\operatorname{sen}\varphi\frac{\partial g}{\partial x} + r\cos\theta\operatorname{sen}\varphi\frac{\partial g}{\partial y}, \\ \frac{\partial u}{\partial \varphi}(r,\theta,\varphi) &= r\cos\theta\cos\varphi\frac{\partial g}{\partial x} + r\operatorname{sen}\theta\cos\varphi\frac{\partial g}{\partial y} - r\operatorname{sen}\varphi\frac{\partial g}{\partial z}, \end{aligned}$$

em que as derivadas parciais com relação a g são calculadas em $(x,y,z) = (r\cos\theta\operatorname{sen}\varphi,\ r\operatorname{sen}\theta\operatorname{sen}\varphi,\ r\cos\varphi)$.

(d)

$$\begin{aligned} \frac{\partial g}{\partial x}(x,y,z) &= \frac{x}{\sqrt{x^2+y^2+z^2}}\frac{\partial u}{\partial r} - \frac{y}{x^2+y^2}\frac{\partial u}{\partial \theta} + \frac{zx/\sqrt{x^2+y^2}}{x^2+y^2+z^2}\frac{\partial u}{\partial \varphi} \\ \frac{\partial g}{\partial y}(x,y,z) &= \frac{y}{\sqrt{x^2+y^2+z^2}}\frac{\partial u}{\partial r} + \frac{x}{x^2+y^2}\frac{\partial u}{\partial \theta} + \frac{zy/\sqrt{x^2+y^2}}{x^2+y^2+z^2}\frac{\partial u}{\partial \varphi} \\ \frac{\partial g}{\partial z}(x,y,z) &= \frac{z}{\sqrt{x^2+y^2+z^2}}\frac{\partial u}{\partial r} - \frac{\sqrt{x^2+y^2}}{x^2+y^2+z^2}\frac{\partial u}{\partial \varphi}, \end{aligned}$$

em que as derivadas parciais com relação a u são calculadas em $(r(x,y,z),\theta(x,y,z),\varphi(x,y,z))$.

(e)

$$\|\nabla g\|^2 = \left(\frac{\partial u}{\partial r}\right)^2 + \frac{1}{x^2+y^2}\left(\frac{\partial u}{\partial \theta}\right)^2 + \frac{1}{x^2+y^2+z^2}\left(\frac{\partial u}{\partial \varphi}\right)^2,$$

em que as derivadas parciais com relação a u são calculadas em $(r(x,y,z), \theta(x,y,z), \varphi(x,y,z))$.

2. (a) $Y'(1) = -\pi/2$.
   (b) $Y'(1) = -1$.

3. (a) $\frac{\partial Z}{\partial x}(0,\pi/2) = -1$ e $\frac{\partial Z}{\partial y}(0,\pi/2) = 0$.
   (b) $\frac{\partial Z}{\partial x}(1,2,8) = 1/3$, $\frac{\partial Z}{\partial y}(1,2,8) = 1/3$ e $\frac{\partial Z}{\partial w}(1,2,8) = -1/6$.
   (c) $\frac{\partial Z}{\partial x}(0,0) = 1$ e $\frac{\partial Z}{\partial y}(0,0) = 0$

4. $\sqrt{3}x - y - z = \frac{\sqrt{3}-2}{6}\pi$.

5. $\theta = 0$.

6. $\frac{\partial s}{\partial x} = F'(t).\frac{\partial g}{\partial x}$ e $\frac{\partial s}{\partial y} = F'(t).\frac{\partial g}{\partial y}$.

7. (a)

$$Jf(x,y) = \begin{bmatrix} e^{x+2y} & 2e^{x+2y} \\ 2\cos(y+2x) & \cos(y+2x) \end{bmatrix}$$

e

$$Jg(u,v,w) = \begin{bmatrix} 1 & 4v & 9w^2 \\ -2u & 2 & 0 \end{bmatrix}$$

   (b) $h(u,v,w) = (\exp(u - 2u^2 + 4v + 2v^2 + 3w^3), \text{sen}(2u - u^2 + 2v + 4v^2 + 6w^3))$.
   (c)

$$Jh(1,-1,1) = \begin{bmatrix} -3 & 0 & 9 \\ 0 & -6\cos 9 & 18\cos 9 \end{bmatrix}$$

8. $f(x) = x^2 - C$, com constante $C \geq 0$.

9.

$$\frac{\partial x}{\partial u} = \frac{xv-1}{x-y}, \quad \frac{\partial x}{\partial v} = \frac{xu+1}{x-y}, \quad \frac{\partial y}{\partial u} = \frac{-yv+1}{x-y}, \quad \frac{\partial y}{\partial v} = \frac{-yu-1}{x-y}.$$

10.

$$\frac{\partial v}{\partial u} = \frac{-yv+1}{1+uy}, \quad \frac{\partial v}{\partial y} = \frac{-x+y}{1+uy}. \quad \frac{\partial x}{\partial u} = \frac{v+u}{1+uy}, \quad \frac{\partial x}{\partial y} = \frac{-xu-1}{1+uy}.$$

11.

$$\frac{\partial f}{\partial x} = \frac{1}{1+2(y+z)}, \quad \frac{\partial f}{\partial y} = \frac{-2(y+z)}{1+2(y+z)}, \quad \frac{\partial^2 f}{\partial x \partial y} = \frac{2}{(1+2y+2z)^3}.$$

12. $\pm(1/\sqrt{751})(-24, 4\sqrt{7}, -3\sqrt{7})$.

13. $\frac{\partial X}{\partial u}(\pi/2, 0) = 0$ e $\frac{\partial X}{\partial v}(\pi/2, 0) = \pi/12$.