Minimal Perfect Hash Functions Based on Random Graphs Containing Cycles

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Objective of the Presentation

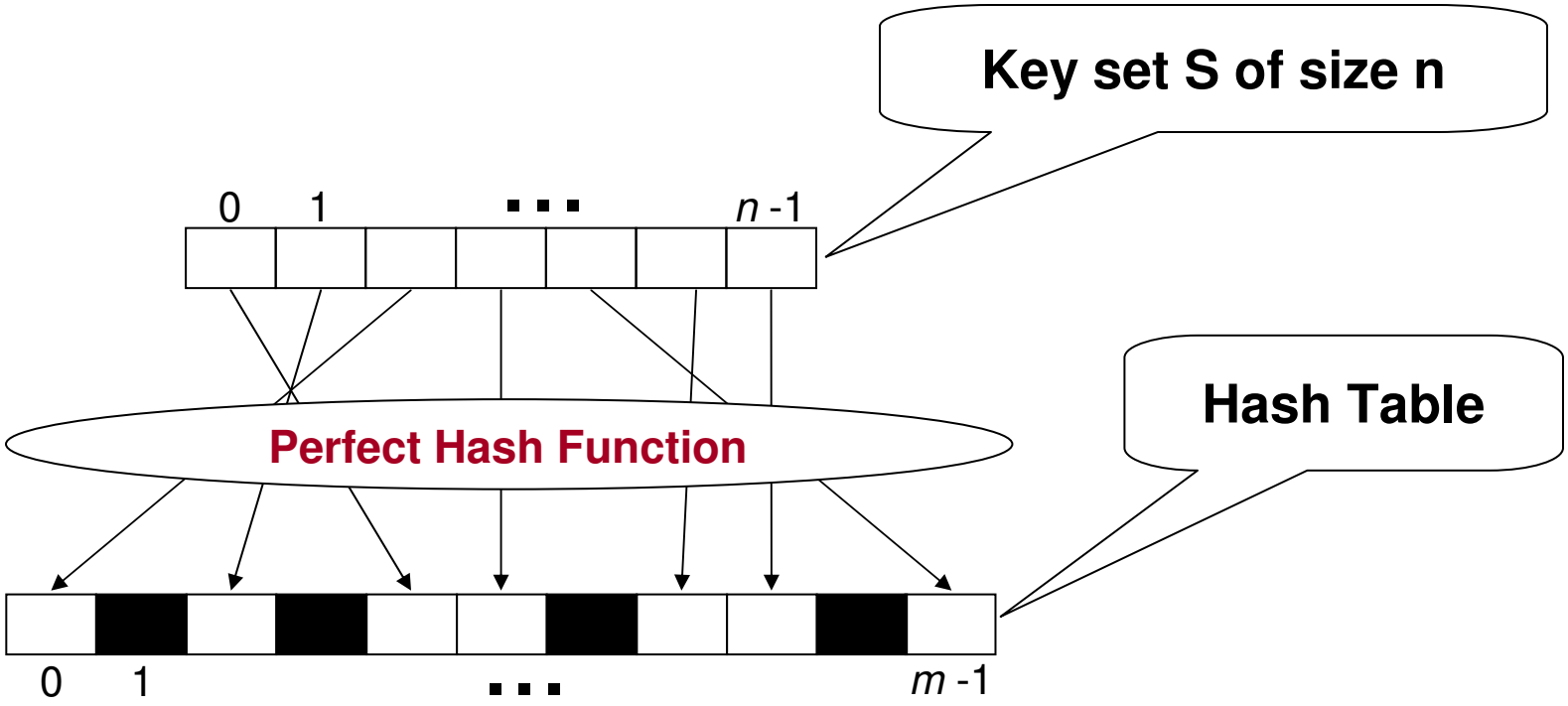
Show how to generate minimal perfect hash functions based on random graphs containing cycles

Some Results

Algorithms	Generation Time(sec)	Storage Space (bits/key)
Czech, Havas and Majewski (IPL, 92)	17.03 ± 3.24	45.47
Botelho, Kohayakawa and Ziviani (WEA,05)	12.83 ± 1.58	21.76
Botelho, Pagh and Ziviani (WADS, 07)	15.04 ± 2.80	3.3
Botelho, Pagh and Ziviani (to be submitted)	10.92 ± 1.18	3.3

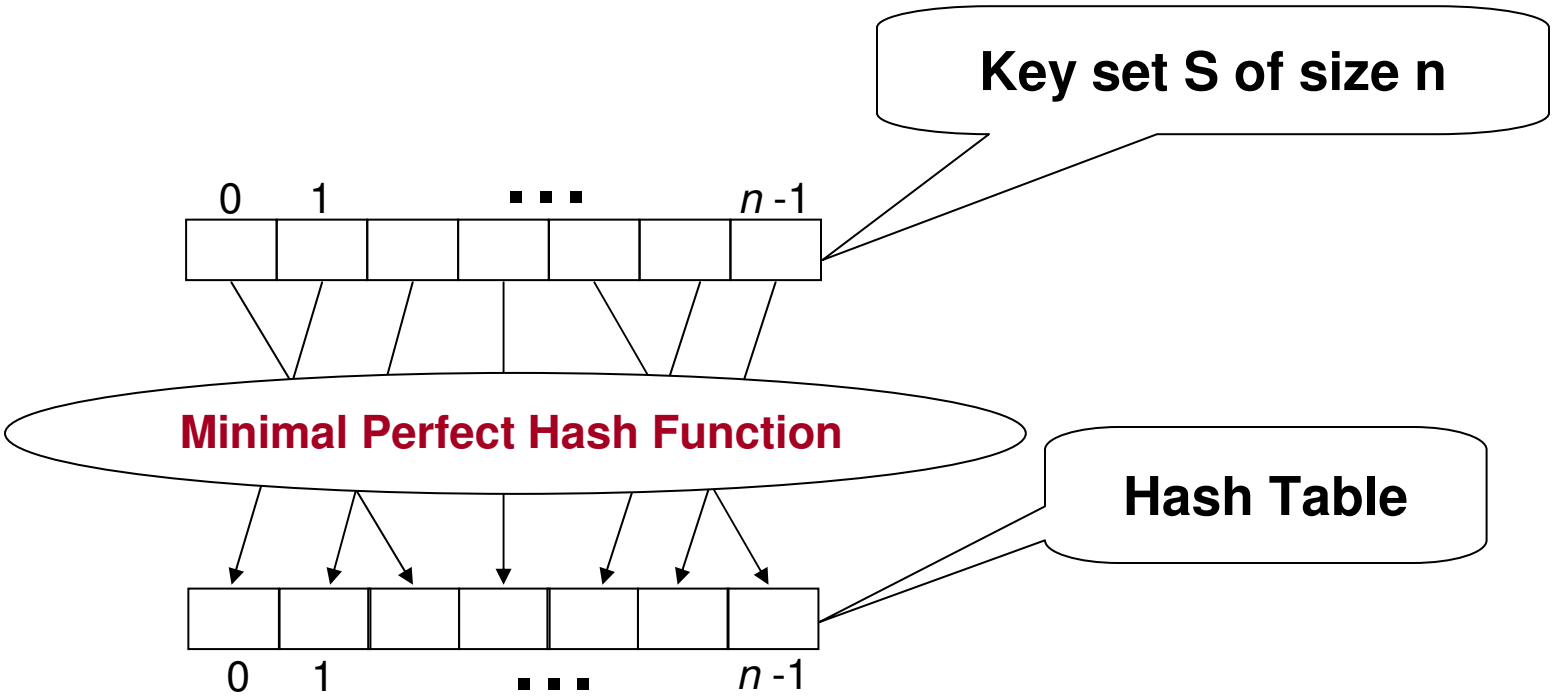
A set of 3,541,615 URLs

Perfect Hash Function



$$S \subseteq U, \text{where} |U| = u$$

Minimal Perfect Hash Function



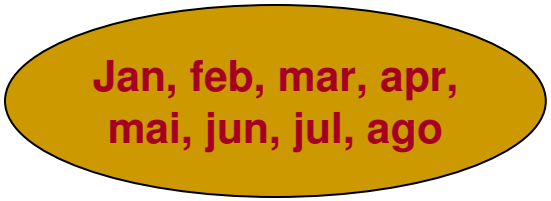
$$S \subseteq U, \text{where} |U| = u$$

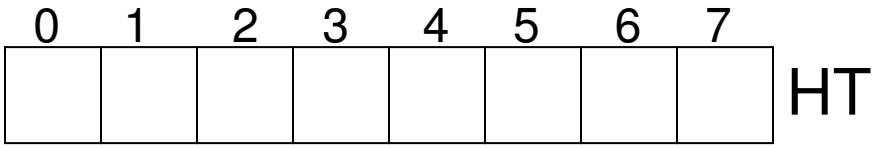
The Algorithm

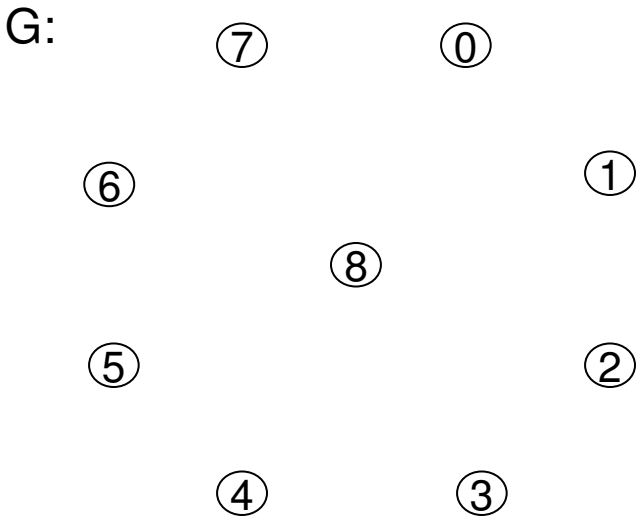
Botelho, Kohayakawa and Ziviani (WEA 2005):

- Improves space requirement of the algorithm by Czech, Havas and Majewski (IPL1992).
- Generates functions stored in O(n log n) bits
- **Is not completely analyzed**

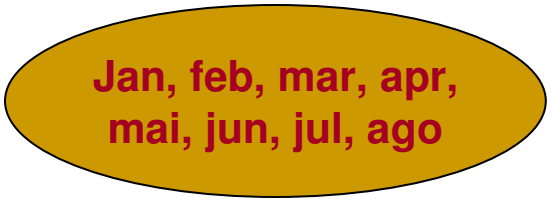
First Step - Mapping

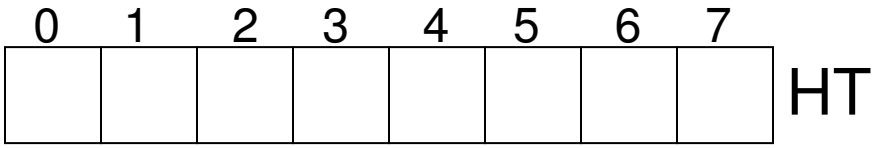




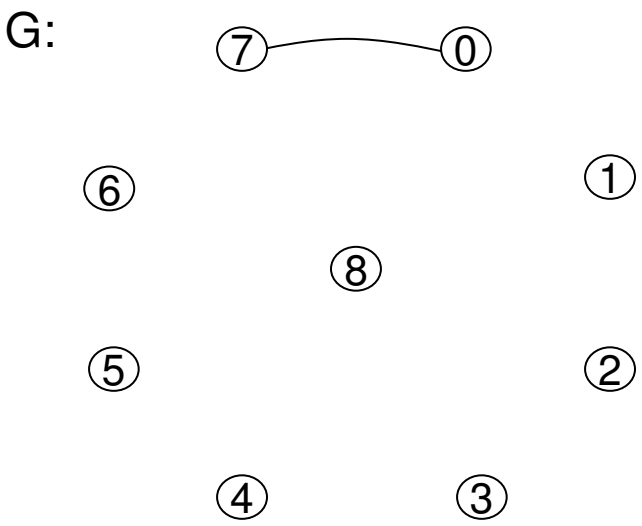


First Step - Mapping

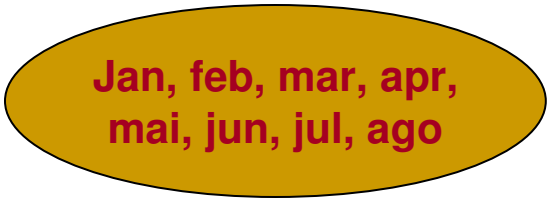


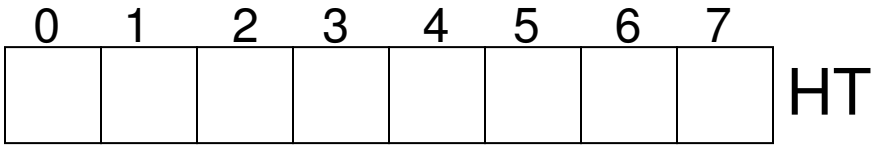


h_1(**jan**) = 7 h_2(**jan**) = 0



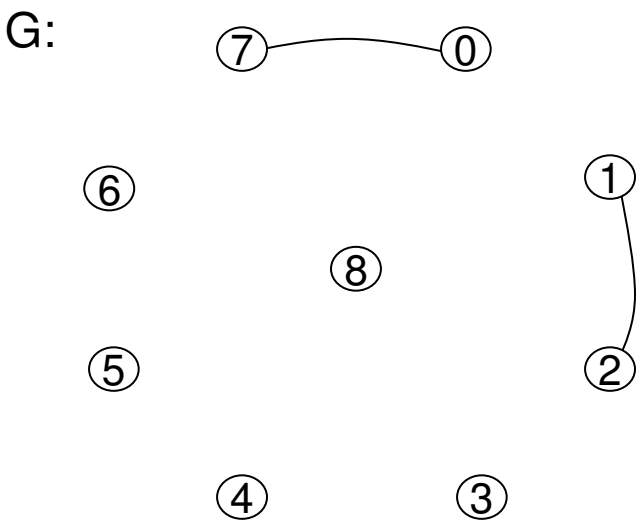
First Step - Mapping





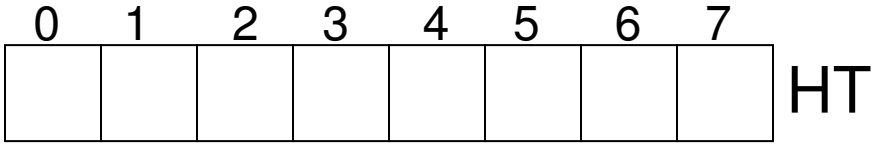
h_1(**jan**) = 7 h_2(**jan**) = 0

h_1(**feb**) = 1 h_2(**feb**) = 2



First Step - Mapping

Jan, feb, mar, apr, mai, jun, jul, ago



G must be simple and must have at most 50% of edges in cycles.

h_1(**jan**) = 7 h_2(**jan**) = 0

h_1(**feb**) = 1 h_2(**feb**) = 2

h_1(**mar**) = 8 h_2(**mar**) = 1

h_1(**apr**) = 3 h_2(**apr**) = 4

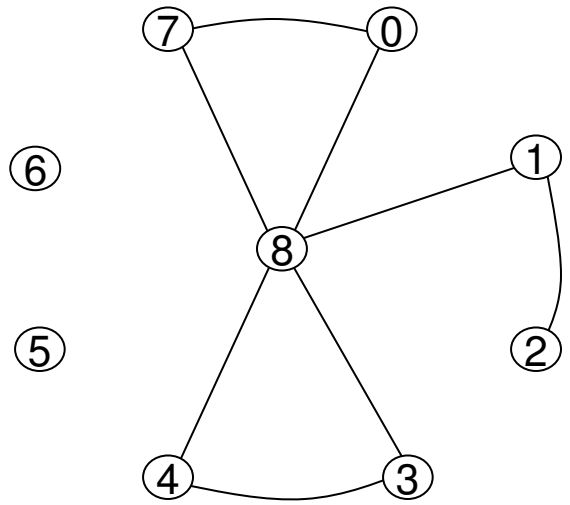
h_1(**mai**) = 4 h_2(**mai**) = 8

h_1(**jun**) = 8 h_2(**jun**) = 0

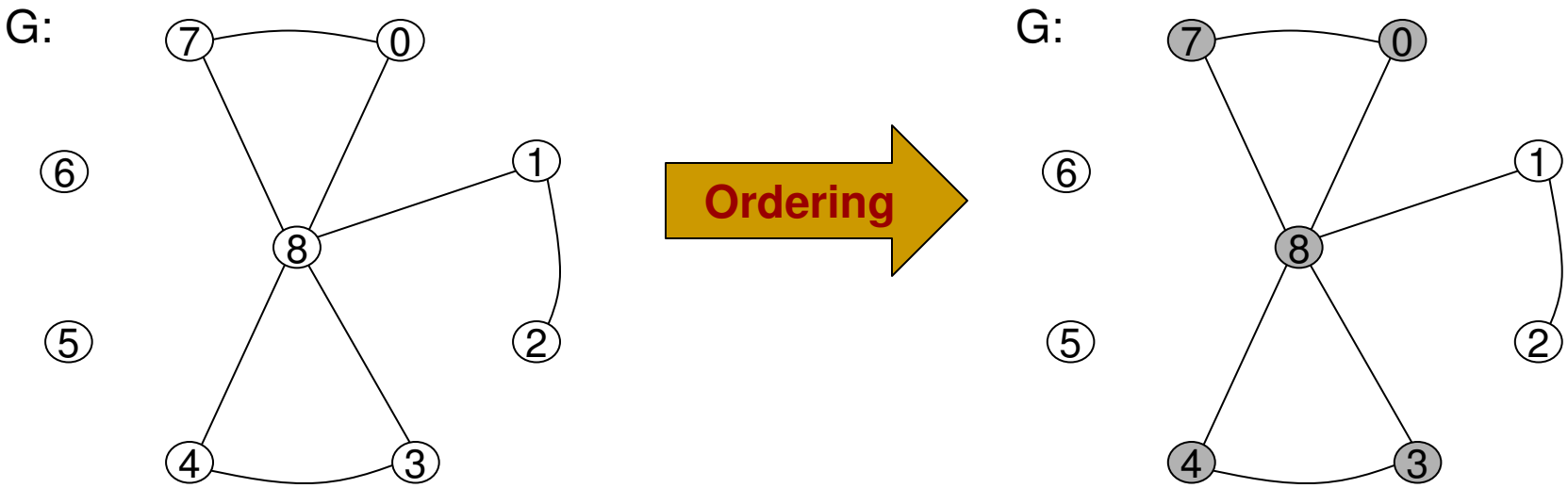
h_1(**jul**) = 3 h_2(**jul**) = 8

h_1(**ago**) = 7 h_2(**ago**) = 8

G:



Second Step - Ordering



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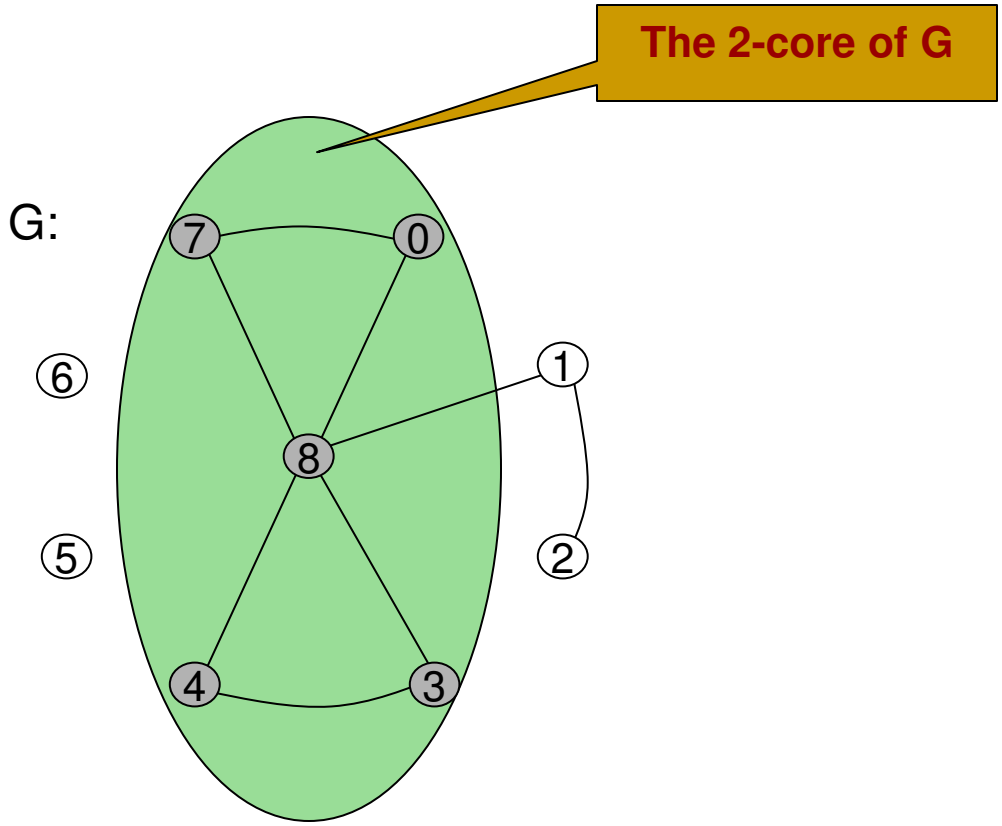
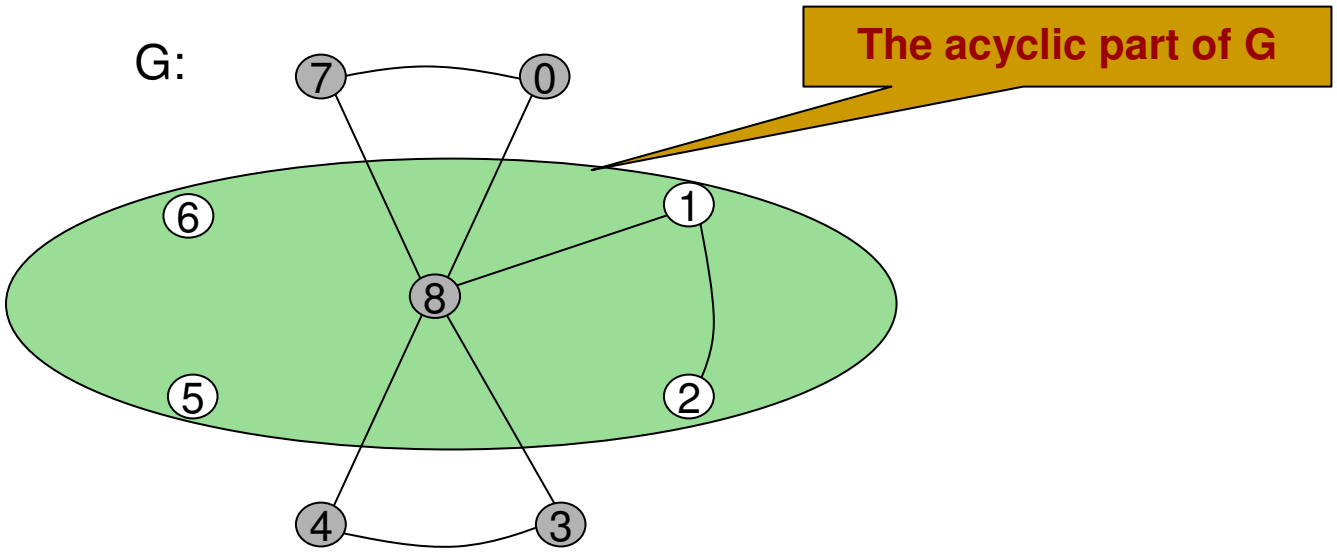


Second Step - Ordering



Third Step - Searching

Assign values to the vertices such that:

$$h(e) = g(a) + g(b)$$

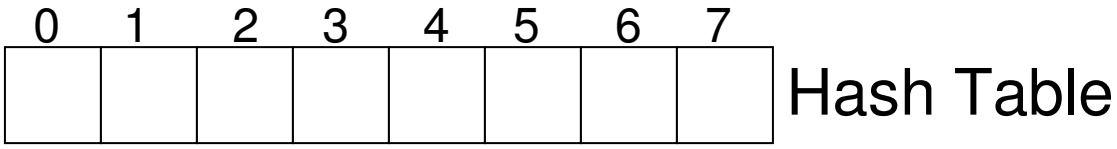
MPHF

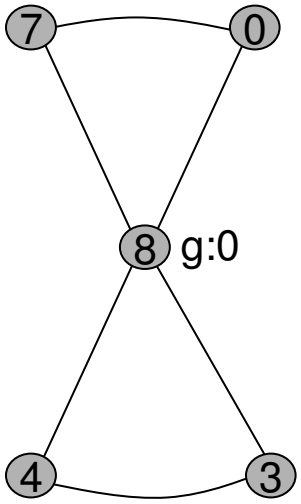
Where:

1. e = {a, b}
2. a = $h_1(x)$
3. b = $h_2(x)$
4. x is in a key of S

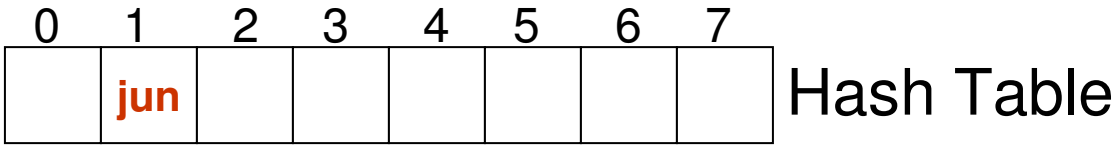
First: Critical part (2-core)
Second: Non critical part (acyclic part)

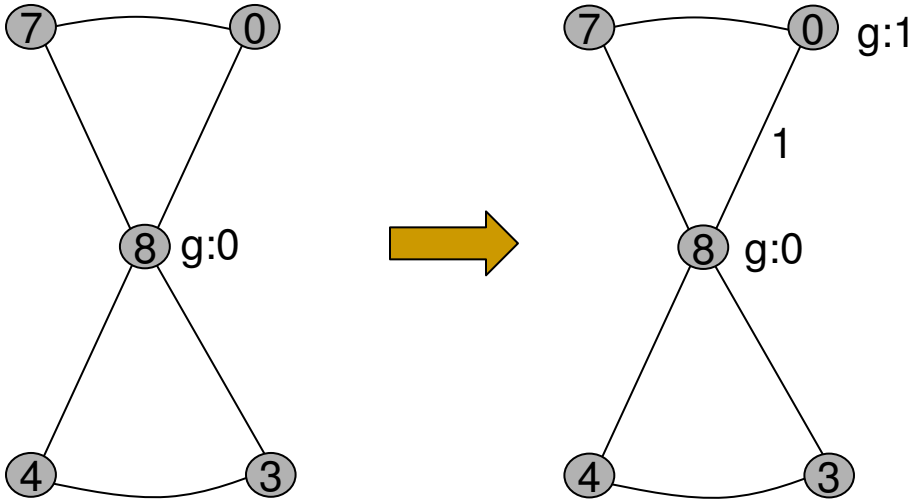
Assignment of Values to Critical Vertices





Assignment of Values to Critical Vertices





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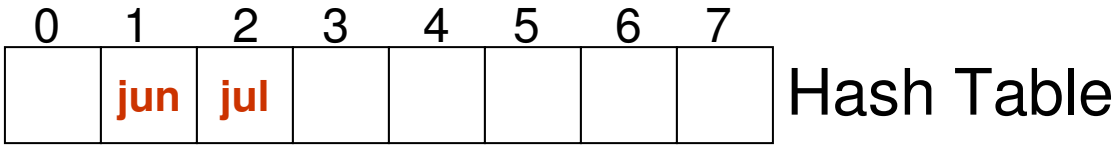
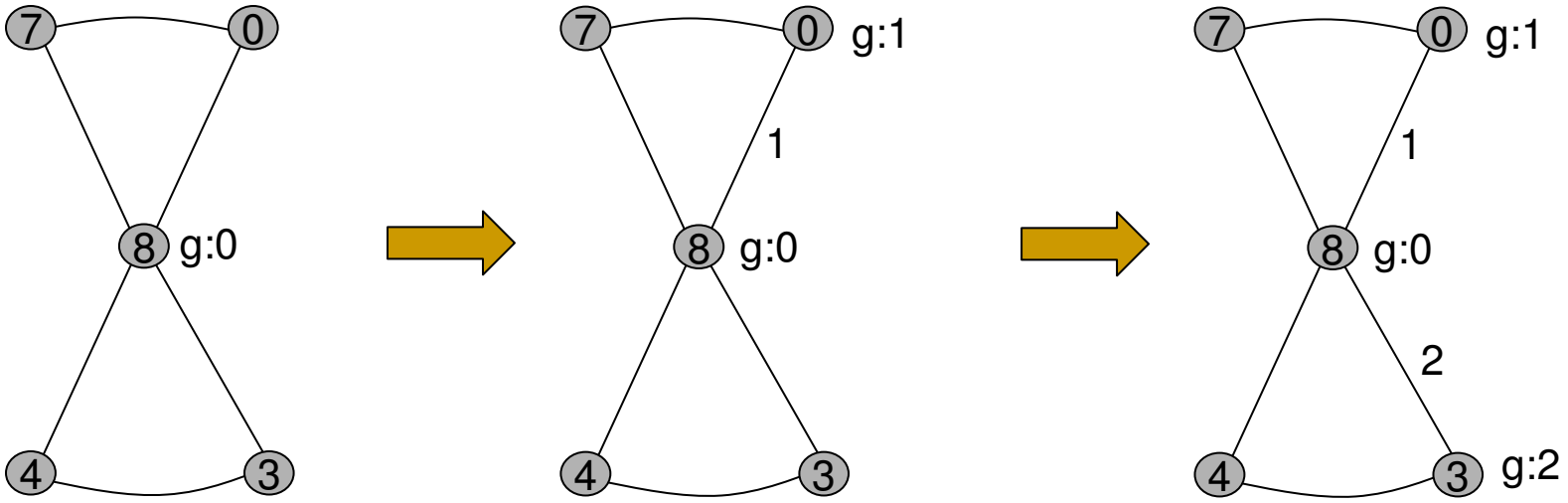
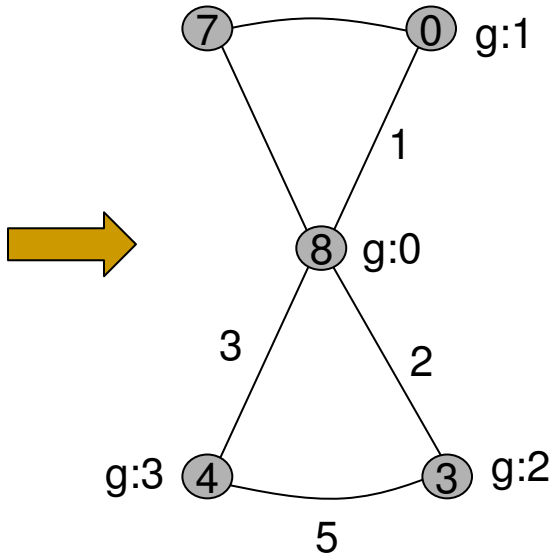




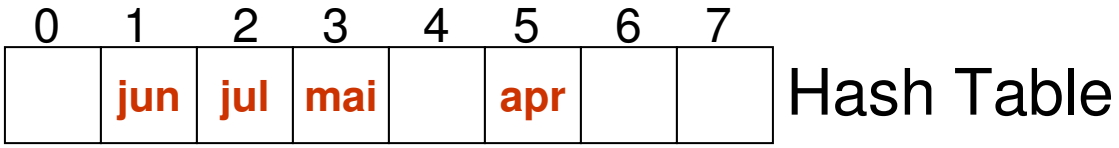
Assignment of Values to Critical Vertices

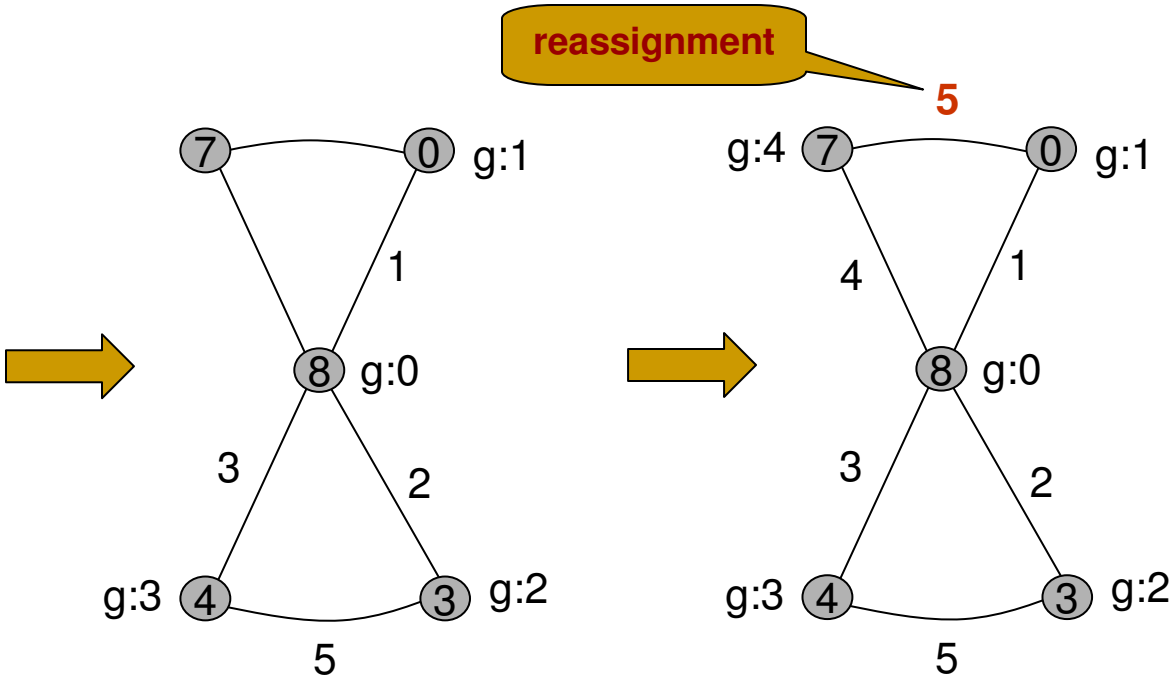
0	1	2	3	4	5	6	7
	jun	jul	mai		apr		

Hash Table



Assignment of Values to Critical Vertices





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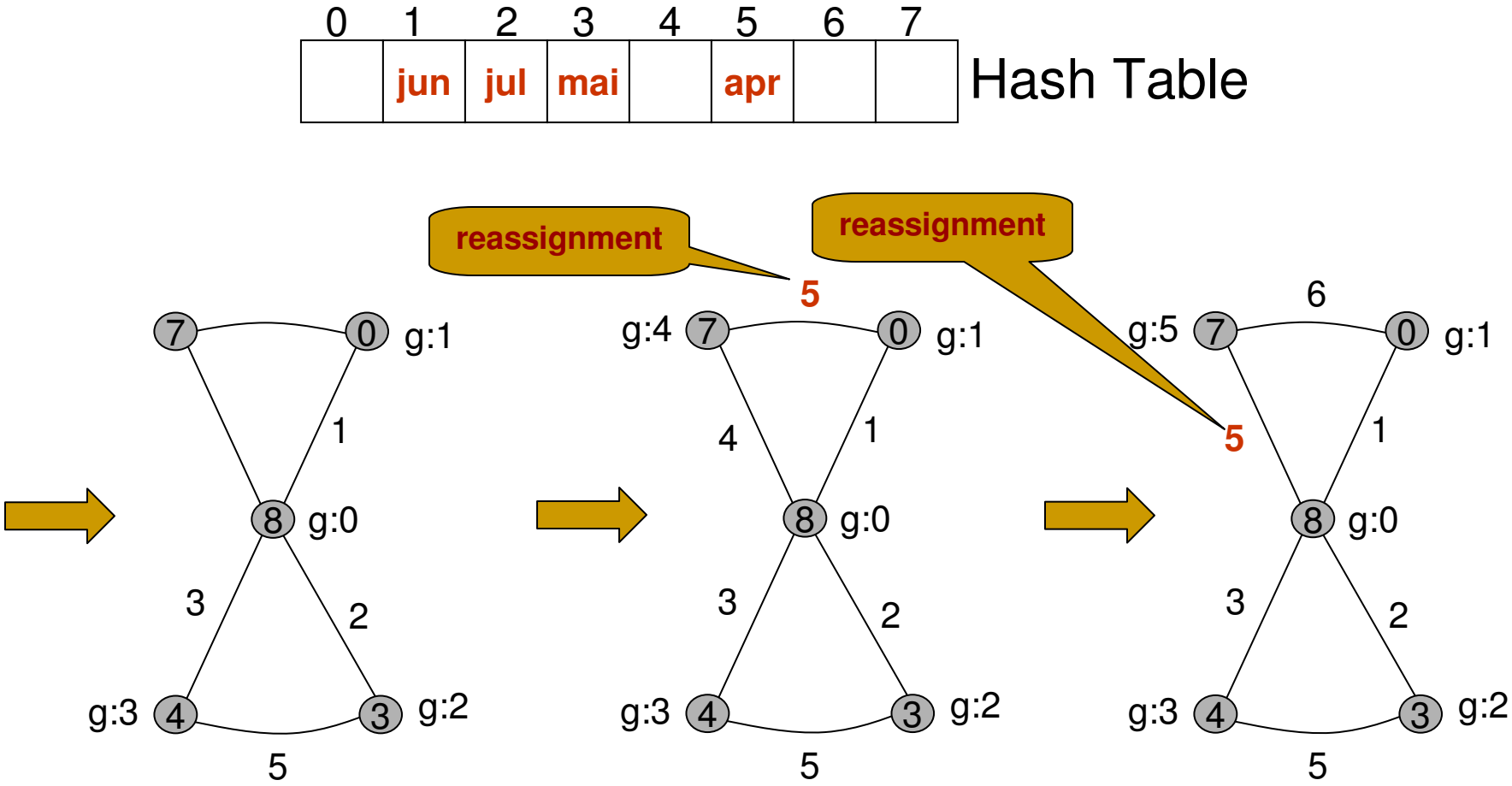
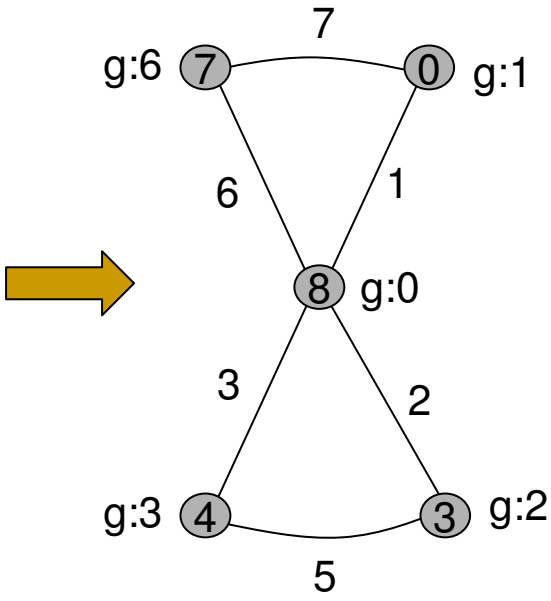


Assignment of Values to Critical Vertices

0	1	2	3	4	5	6	7
	jun	jul	mai		apr	ago	jan

Hash Table



Used addresses: {1,2,3,5,6,7}

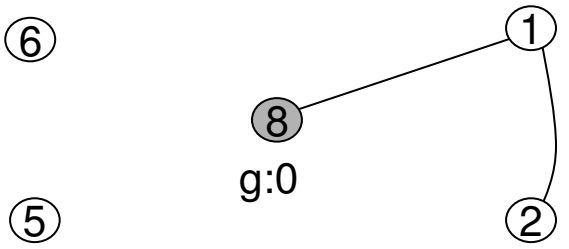
Unused addresses: {0,4}

Assignment of Values to Non-Critical Vertices

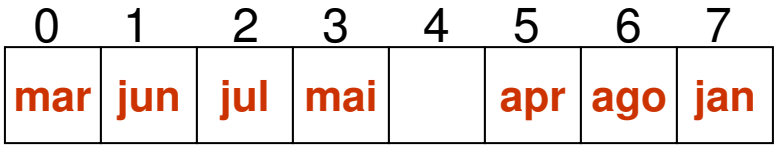
0	1	2	3	4	5	6	7
	jun	jul	mai		apr	ago	jan

Hash Table

Unused addresses: {0,4}

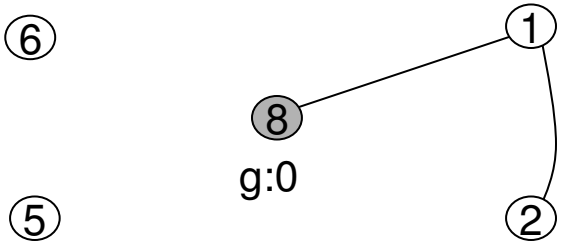


Assignment of Values to Non-Critical Vertices

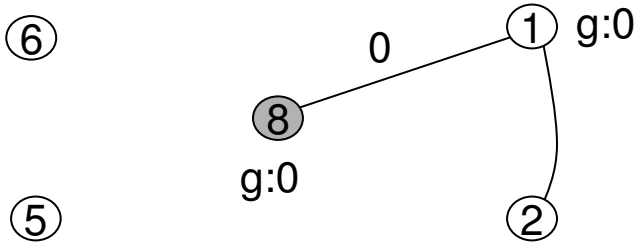


Hash Table

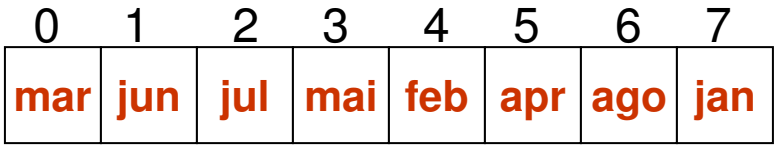
Unused addresses: {0,4}



Unused addresses: {4}

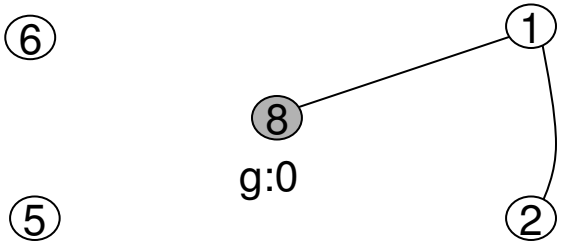


Assignment of Values to Non-Critical Vertices

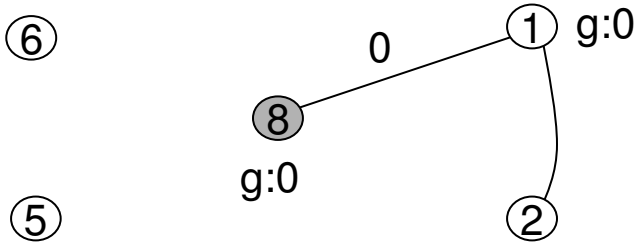


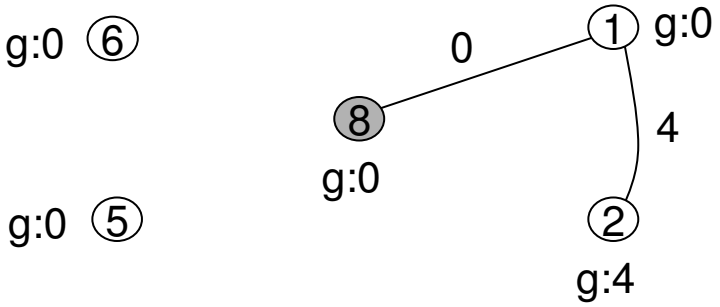
Hash Table

Unused addresses: {0,4}



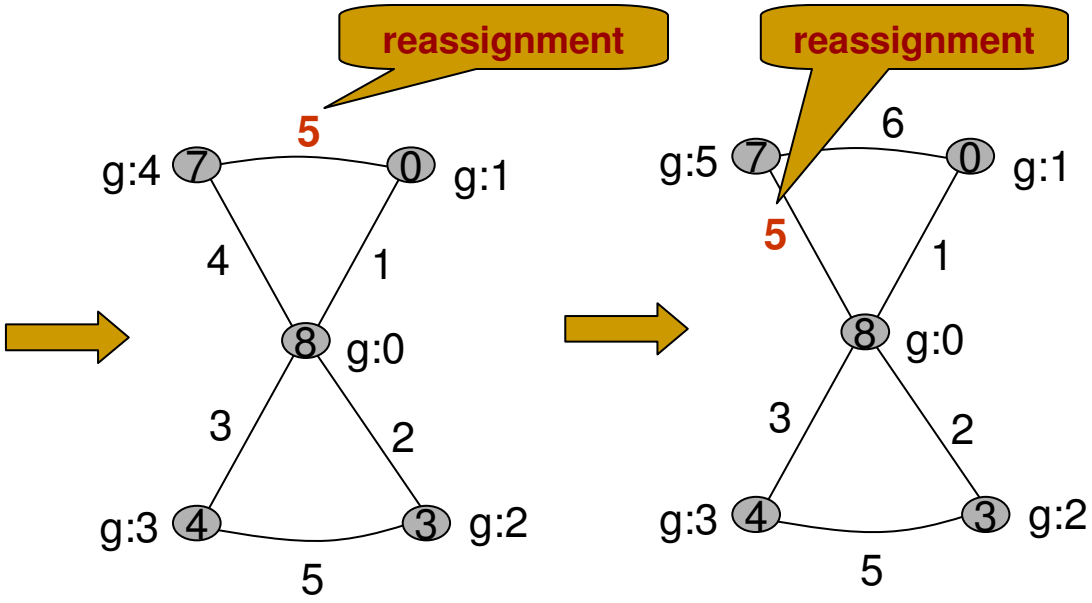
Unused addresses: {4}





Unused addresses: {}

The Open Problem



- The maximal value assigned to an edge is:

$$A_{max} \leq 2 | V_{crit} | - 3 + 2N_t$$

- The number of back edges of G is:

$$N_{bedges} = |E_{crit}| - |V_{crit}| + 1$$

The Open Problem

- Conjecture: $N_t \leq N_{bedges}$

$$A_{max} \leq 2|V_{crit}| - 3 + 2N_{bedges}$$

$$A_{max} \leq 2|V_{crit}| - 3 + 2(|E_{crit}| - |V_{crit}| + 1)$$

$$A_{max} \leq 2|E_{crit}| - 1$$

- If $|E_{crit}| \leq 0.5n$ then $A_{max} \leq n - 1$ and a MPHF is generated in linear time.

- The only problem is left open is:

Prove that $N_t \leq N_{bedges}$

Experimental Evidences

- Experimental evidences that $N_t \leq N_{bedges}$:
- Recall: $N_{bedges} = |E_{crit}| - |V_{crit}| + 1 = 0.501n - 0.401n + 1 = 0.1n + 1$.

n	Expected N_{bedges}	Maximal Value of N_t
10,000	0.1n + 1	0.067n
100,000	0.1n + 1	0.061n
1,000,000	0.1n + 1	0.059n
2,000,000	0.1n + 1	0.059n
...	...	...

Approach Used for Constructing Minimal Perfect Hash Functions

- MOS - Mapping, Ordering and Searching:
  - **Mapping**: transforms the key set from the original universe to a new universe.
  - **Ordering**: places the keys in a sequential order that determines the order in which hash values are assigned to keys.
  - **Searching**: attempts to assign hash values to the keys.

At Most 50% of Edges in the 2-Core

- The number of vertices is a linear function on the number of edges: |V(G)|=cn
- Pittel and Wormald (2005), present detailed results for the 2-core of the giant component of a random graph G.

$\|V_{crit}\|$	$\|E_{crit}\|$	c
0.399n	0.498n	1.153
0.400n	0.500n	1.152
0.401n	0.501n	1.151
0.401n	0.501n	1.150
0.401n	0.502n	1.149

- We determined empirically that c = 1.15.